

AGI and HSA (Hypertopology/Semigroup/Acyclicity)

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ABSTRACT.

AGI stands for artificial general intelligence and HSA stands for 3 areas (hypertopology/semigroup/acyclicity) in mathematics. The author will describe the connection between the two fields.

In 1974, the author published a paper on "Acyclicity and dimension of hyperspaces of subcontinua" in Buletin de l'Académie Polonaise des Sciences" (Bulletin of the Polish Academy of sciences). The paper proved that certain hyperspaces are acyclic (no holes in the hyperspace i.e. all Betti numbers are zero) and are often infinite dimensional. It used topological semigroup theory (specifically semilattice) to prove the acyclicity of certain hyperspace. That was the paper that connected three areas of abstract mathematics:

Hypertopology

Semigroup

Acyclicity.

HYPERTOPOLOGY:

Hypertopology is the contraction of two words hyperspace and topology. If X is a set (sometimes called a base space), then the hyperspace is the collection of certain subsets of X . So if $X = \{1,2\}$, then $H(X)$ can be defined as the hyperspace of X composed of $\{1\}$ and $\{2\}$ and $\{1,2\}$. Hyperspace has been used to just indicate higher dimension like 10 compared to 4 dimension. In this note, hyperspace is a "construction" which collects certain subsets of a base space X .

Why topology? A base space needs to be endowed with a topology if we want the concept of abstract distance or abstract geometry to be used on the base space. when measurements are made, the open sets of the topology of X is an abstract description of proximity.

One connection to AGI: The more narrow definition of hyperspace has been used in AGI as hypergraphs connecting points. The points are connected by edges (connecting two points) and graphs (connecting more than two points).

SEMIGROUP:

A semigroup is endowed with an abstract binary associative operation (binary operation of combining x and y is xy) by combining x and y in base space X (e.g. $xy=x+y$, $xy=\min(x,y)$, square matrix multiplication, etc. The associative operation requires $x(yz)=(xy)z$. The base space can be endowed with a semigroup structure to denote abstract algebra. Classical use of semigroup is to view semigroup as a semigroup of operators. Semigroup is a collections of actions on a space.

Why semigroup? Semigroup is a generalization of group theory. Group theory requires an inverse which means if you move forward, then you can move backward to the original spot. Many real world problems are not reversible. One example is time is irreversible. Time goes forward and loses information. Instruments are never so perfect that nothing is lost as time goes on. So semigroup could be viewed as dynamics that are not necessarily reversible.

Semilattice is a special kind of semigroup which is a commutative idempotent semigroup. Idempotent means $xx=x$ for every x in X and commutative means $xy=yx$ for any x,y in X . The 1974 used compact semilattice to prove certain hyperspaces are acyclic (no cycles, no holes, Betti numbers zero). The hyperspace has a "data structure" similar to semilattice where each element (subset) with "multiplication" being union of two subsets. For general data structure, if it is a semilattice, then it creates a partial order of the elements based on the semilattice binary operation.

ACYCLICITY:

Acyclicity is an important concept about having no holes (i.e. no loops, zero Betti numbers). Semilattice was used in the 1974 paper to prove acyclicity of certain hyperspace which is often times infinite dimensional. Hence hyperspace has a lot of room to move a subset A to subset B without obstructions like holes. Even though the hyperspace is acyclic, each subset

could be non-acyclic. Acyclicity is an important property in using hypergraph for AGI. Graphs that have holes could have loops that double back many times. This could create many or forever loops that go nowhere and get stuck.

Another reason for acyclicity is that we want dynamics to not double back. For wave equation, we want a one-way wave equation so it will not reverberate. We know that real world wave does double back. But in an artificial world like AI, we can accept only acyclic dynamics. In this way, it creates uniqueness so that anything that goes against one way will be flagged as attacks or hacking. It means the code cannot double back which seems like a constraint. But it also means more restrictive syntax which distinguishes "our syntax" different than other programs. This harks back to the 1930s concept of lambda calculus.

SUMMARY:

Even though this is a high level note on hyperspace, semigroup and acyclicity, the author hopes that this note will encourage using more abstract concepts that could lead to a more systematic approach to AGI.

PATHFORWARD:

Find the hyperspace appropriate for specific AGI.

Define the semigroup of operators (actions on objects or other semigroups).

Define acyclicity in objects by defining its topology.

MISCELLANEOUS:

Maybe of interest in a big picture sense.

"The Unreasonable Expectation of Mathematics", 2018, viXra and "The End of Mathematics", 2019, viXra were written to warn of the limitation of mathematics and to know the purpose (the end) of mathematics.

The paper on LG space is to warn of the difficulty of LG (locally dense and

globally sparse) for inverse problems in seismic acoustic imaging. This paper is related to "Seismic Solvability Problems", 2012, arXiv. It pointed out the strangeness of hyperspace. If C is the totally disconnected Cantor set in the unit interval $[0,1]$ and I is the continuous interval $[0,2]$, then $C+C = I$. It shows that all combinations of $a+b$ for arbitrary a and b in C (no connected piece in Cantor set) will fill up the continuum $[0,2]$ which is one connected piece.

Here is the list of the author's publications:

Part 1 : Academic Publications

1. Small semilattices, *Semigroup Forum*, 4 (1972)
2. Concerning Costability of compact semigroups, *Duke Math Journal*, No.4, Vol 39 (1972)
3. Coverable semigroups, *Proceedings of the American Math Society*, No.3, Vol 38 (1973)
4. Costability in SEM and TSL, *Semigroup Forum*, Vol 5 (1973)
5. Problems 44 and 39 on Compact semilattices, *Semigroup Forum*, Vol 9 (1974)
6. Acyclicity and dimension of hyperspaces of subcontinua, *Bulletin Polish Acad. Sci.*, No. 11, Vol XXII (1974)
7. Codimension of compact M-semilattices, *Proceedings of the American Math Society*, Vol 52 (1975) (with Lea)
8. The boundary of a semilattice of an n-cell, *Pacific Journal of Math*, No. 1, Vol 56 (1975)
9. Open maps on totally disconnected spaces and groups, *Nanta Math*, No. 1, Vol X (1975)
10. A note on monotone maps and hyperspaces, *Bulletin Polish Acad. Science*, No. 2, Vol XXIV (1976)
11. Existence of n-cells in Peano semilattices, *Topology*, Ed. S.P. Franklin and B.V.S. Thomas, Marcel Dekker Publishing, Vol 24 (1976)
12. Whitney continuum in hyperspaces, *Topology Proceedings*, Vol 1 (1976)
13. Finitely neighborable groupoids, *Kyungpook Math Journal*, No.2, Vol 16

(1976)

14. Representations into the hyperspaces of a compact group, Semigroup Forum, Vol 13 (1977) (with Bilyeu)
15. Connectedness of hyperspaces of closed connected subsets, Commentationes Mathematicae, Vol XX (1978) (with Voas)
16. Certain local homeomorphisms of continua are homeomorphisms, Bulletin Polish Acad Science, No. 4 Vol XXVI (1978)
17. Quotients of compact 0-dimensional semigroups and groupoids, Semigroup Forum, Vol 16 (1978)
18. Finite abelian semigroups represented into the power of finite groups, Volume 29, Czechoslovak Math Journal (1979)
19. Images of compact 0-dimensional semigroups, Colloquium Mathematicum, No. 2, Vol XL (1979)
20. Certain local homeomorphisms of continua are homeomorphisms, II, Bulletin Polish Acad. Science, No.5, Vol XXVII (1979)
21. Plane continua and transformation groups, Proceedings American Math Society, No.4 Vol 78 (1980)

Part 2 : Post-academic Publications

1. A Post-Stack Method for 3-D Crossline Statics Estimation, SEG Annual Meeting 1982 (with P. Schultz)
2. Poststack estimation of three-dimensional crossline statics, Geophysics, Vol 49 March 1984 (with P. Schultz)
3. Common-Offset Processing with DMO-Driven Statics, SEG Annual Meeting 1991 (with Wilmott)
4. Optimal Multiple Extrapolation and Attenuation, SEG Annual Meeting 1997 (with Pham)
5. Two-step approach for depth imaging: an application in the Western Desert, Egypt Exploration Conference 1998 (with Bakr et al)
6. Tomography in Egypt - 3D depth imaging improvements in the Qarun Field Western Desert, EAGE Annual Meeting 1999 (with Bakr et al)
7. Unconstrained near-surface velocity inversion of linear moveout velocities and its impact on structural mapping, SEG Annual Meeting 1999 (with Gonzalez et al)

8. A comparison of direct and indirect near-surface velocity measurement for velocity model building and prestack depth migration in an area of shallow carbonate overburden, ASEG Annual Meeting 2000 in Australia (with Hansen et al)
9. Multiple prediction for velocity analysis, SEG Annual Meeting 2000 (with Gasparotto)
10. Tomostatics: recent development and applications, SEG Annual Meeting 2001 Road Ahead Presentation (with Zhu et al)
11. Prediction of resistivity from seismic : an integrated study of reprocessing, well logs, neuralnet and prospecting, SEG Annual Meeting 2001 (with Yin)
12. Full waveform prestack inversion and neural network interpolation, EAGE Annual Meeting 2002 (with Gonzalez et al)
13. Tomography for near-surface and imaging velocity determination - Examples from the Western Desert in Egypt, Cairo 2002 Conference and Technical Exhibition (with Bedingfield et al)
14. Waveform gather inversion and attribute-guided interpolation: A two-step approach to log prediction, The Leading Edge, Oct 2002 (with Gonzalez et al)
15. Building High-Resolution Velocity Models in the Gulf of Mexico with wave-equation Depth Migration - SEG Annual Meeting 2003 (with Popovici et al)
16. Seismic prediction and micro-tomography; a case study of small travelttime error and waveform distortion, SEG Annual Meeting 2003 (With Yin)
17. Colored Inversion: Application in a Tertiary Basin Offshore China, OTC Houston 2005 (with Dai et al)
18. Hybrid inversion techniques used to derive key elastic parameters: A case study from the Nile Delta , The Leading Edge, Jan 2005 86-92 (with Roberts et al)
19. Prestack waveform inversion in the western Nile delta, SEG Annual meeting 2005 (with Roberts et al)
20. Complex Topology : its impact on seismic inversion and modeling in single component recording and multicomponent recording, SEG Annual Meeting 2005 (with Yin)
21. Imaging Beneath Complex Layering : Processing seismic data with

- complex decomposition and renormalization group, SEG Annual Meeting, 2007 (with Lapilli et al)
22. Prestack Waveform Inversion: an Onshore Application in the US Gulf Coast, SEG Annual Meeting 2007 (with Yin et al)
23. Repeat Imaging using grid tomography and new well information: application to two sub-salt fields in the Gulf of Mexico, SEG Annual Meeting 2007 (with Stewart et al)
24. Time-lapse simulator-to-seismic study - Forties field, North Sea, SEG Annual Meeting 2007 (with Ribeiro et al)
25. Imaging with complex decomposition: Numerical applications to seismic processing in difficult areas, SEG Annual Meeting 2008 (with Roque-Sol et al)
26. Diffusion semigroups: A diffusion-map approach to nonlinear decomposition of seismic data without predetermined basis, SEG Annual Meeting 2009 (with Yin et al)
27. Geometric simplicity as a migration criterion: an application of computational topology to seismic imaging, SEG Annual Meeting 2009 (with Yin)
28. $L_0+L_1+L_2$ mixed optimization: a geometric approach to seismic imaging and inversion using concepts in topology and semigroup, arXiv, 2010 (with Yin)
29. Transformation Semigroup and Complex Topology: a study of inversion with increasing Complexity, arXiv, 2010 (with Yin)
30. Practical approach to solvability: Geophysical application using complex decomposition into simple part (solvable) and complex part (intepretable) for seismic imaging, arXiv, 2010 (with Yin)
31. Solvability by semigroup : Application to seismic imaging with complex decomposition of wave equations and migration operators with idempotents, arXiv, 2011 (with Yin)
32. Seismic Solvability Problems, arXiv, 2012 (with Yin)
33. Benefits of constraints for velocity modeling a fault shadow: A case study, SEG Annual Meeting 2012 (with Quigley et al)
34. Seismic depth imaging from high to low signal: a case history , SEG Annual Meeting 2013 (with Yin et al)

35. Short-period multiple attenuation in imaging domain: Use of primaries to predict multiples using complex decomposition and modeling, EAGE 2014 (with Tang et al)
36. Automated travel time tomography using early-arrival Gaussian packets, SEG Annual Meeting 2014 (with Tang et al)
37. Geometric theory of inversion and seismic imaging , arXiv, 2015 (with Yin)
38. Prestack and poststack wave-equation based demultiple: a case study of its impact on subsalt interpretation, SEG Annual Meeting 2015 (with Barker et al)
39. Deep-water fold-and-thrust imaging: A case study with joint inversion, tomography and dense velocity analysis, SEG Annual Meeting 2016 (with Clare et al)
40. Joint inversion for microseismic event positions and velocity structures by combining multi-scale deformable-layer tomography and master station earthquake location method, SEG Annual Meeting 2016 (with Wei et al)
41. Relaxed flatness-constraint of angle-domain-common-image-gathers generating by Poynting vectors via concurrent vector-median-filter, SEG Annual Meeting 2017 (with Pongthep Thongsang et al)
42. Geometric theory of inversion and seismic imaging II: INVERSION + DATUMING + STATIC + ENHANCEMENT 2017, viXra (with Yin)
43. Quantitative and Qualitative Seismic Imaging and Seismic Inversion arXiv 2017 (with Yin)
44. Direct and indirect seismic inversion: interpretation of certain mathematical theorems arXiv 2017 (with Yin)
47. Imaging improvement in angle-domain common-image-gathers by a local stack utilizing segmentation method, SEG 2018 (with Pongthep Thongsang et al)
48. The Unreasonable Expectation of Mathematics, 2018, viXra
49. The End of Mathematics, 2019, viXra
50. Identifying imaging challenges for 1D and 3D geometric complexity (LG space), 2024, The Leading Edge (with A. Gonzalez)
51. AGI and HSA (Hypertopology/Semigroup/Acyclicity), 2026, viXra