

Torsion in disguise: scalar-gradient contortion, bosonic triviality, and the spin coupling that survives

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When a geometric modification of spacetime is equivalent, by a field redefinition, to a theory we already know, does anything physical survive the equivalence? We examine a metric-compatible contortion of the form $K^\lambda_{\mu\nu} = F(\phi) \epsilon^{\lambda\rho}_{\mu\nu} \partial_\rho \phi$, generated by the gradient of a scalar field and totally antisymmetric in its three indices, which is unique within its class once one requires that spinless particles follow the geodesics of general relativity. The Ricci scalar acquires a correction $R(\Gamma) = R(\{ \}) + 6F^2(\partial\phi)^2$ that shifts the scalar kinetic term by a factor $1 - 6F^2/\kappa_g$, yielding a ghost bound $M_F > 0.49 M_{\text{Pl}}$, and the Schwarzschild metric remains an exact vacuum solution when the scalar field is constant. We show that the bosonic sector is equivalent, by a field redefinition, to a canonical scalar field minimally coupled to general relativity, so that the modification of the Kalb-Ramond duality is a statement about the parametrization rather than about the physical spectrum. The survival of a residual coupling in the matter sector is not specific to this construction, since it also occurs in the Jordan-Einstein frame equivalence of scalar-tensor theories, but the form of the surviving coupling is. Here the residual coupling is parity-odd, spin-dependent, and non-universal, affecting only matter with spin. We estimate the magnitude of this force in a cosmological background and find it to be far below observational sensitivity; we do not claim that it becomes observable in regimes that we have not calculated.

Keywords: modified gravity; torsion; metric compatibility; scalar-tensor theories; spin-torsion coupling

I. INTRODUCTION

General relativity (GR) has passed every classical test devised so far [1–3], and any proposed extension must reproduce these results before it can claim to predict anything new. Torsion-based extensions illustrate the difficulty of this requirement. Einstein-Cartan theory promotes torsion to an independent field sourced by spin density [4, 5] and modifies the field equations even in vacuum when spin is present. Chern-Simons gravity adds a parity-violating term $\theta R\tilde{R}$ to the action [6, 7]; the Schwarzschild solution survives, but the modified field equations contain third derivatives and raise concerns about Ostrogradski instabilities [8]. Teleparallel gravity reformulates GR in terms of torsion but remains equivalent to it at the level of equations of motion [9].

This note examines a construction in which the contortion is determined entirely by the gradient of a scalar field and is totally antisymmetric in its three indices. The ansatz is

$$K^\lambda_{\mu\nu} = F(\phi) \epsilon^{\lambda\rho}_{\mu\nu} \partial_\rho \phi, \quad (1)$$

with $F(\phi)$ a coupling function of dimension M^{-1} and $\epsilon^{\lambda\rho}_{\mu\nu}$ the Levi-Civita tensor. The corresponding connection is $\Gamma^\lambda_{\mu\nu} = \{\lambda_{\mu\nu}\} + K^\lambda_{\mu\nu}$. The construction has four properties that we develop in this note. First, it is the only non-trivial contortion linear in $\partial_\rho \phi$ that is

totally antisymmetric in its three indices, and this property simultaneously guarantees that spinless particles follow geodesics of the Levi-Civita connection. Second, the Schwarzschild metric remains an exact vacuum solution of the theory when the scalar field is constant. Third, the Ricci scalar acquires a correction $+6F^2(\partial\phi)^2$, which shifts the scalar kinetic term and yields a ghost bound on the coupling. Fourth, the coupling to spinning matter survives a field redefinition that brings the bosonic sector to canonical form, and this coupling is the only physical content of the construction that is not shared with a free scalar field.

We emphasize from the outset that the bosonic sector of the theory is equivalent, by a field redefinition, to a canonical scalar field minimally coupled to GR. The modification of the Kalb-Ramond duality that we discuss in Sec. V is therefore a statement about the parametrization of the theory rather than about the existence of new degrees of freedom. We also note that the survival of a residual coupling in the matter sector after the bosonic trivialization is a generic feature of equivalences between geometric modifications of gravity and known scalar-tensor theories, as illustrated by the Jordan-Einstein frame equivalence of Brans-Dicke theory and by the equivalence between $f(R)$ gravity and Brans-Dicke theory with $\omega = 0$. What distinguishes the present construction is the form of the surviving coupling, which is parity-odd, spin-dependent, and non-universal, in contrast with the conformal and universal coupling of the standard scalar-tensor equivalences. The physical consequences of the surviving coupling are estimated in Sec. VI and found to be far below observational sensitivity in the

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regime that we calculate. We do not claim that the effect becomes observable in regimes that we have not modeled, and we present the paper as a technical note on the structure of the construction.

The note is organized as follows. Section II establishes the framework and proves uniqueness of the ansatz. Section III shows that spinless particles follow Levi-Civita geodesics and that the Schwarzschild metric is an exact vacuum solution. Section IV computes the Ricci scalar, the effective action, and the ghost bound. Section V shows that the bosonic sector is reducible to canonical form and identifies the spin coupling as the invariant content. Section VI derives the spin-torsion force from the equations of motion for a spinning particle in a spacetime with torsion. Section VII compares the construction with Einstein-Cartan, Chern-Simons, and teleparallel gravity. Section VIII summarizes the results.

We work in natural units $\hbar = c = 1$ with metric signature $(-, +, +, +)$. The Riemann tensor is $R^\lambda_{\mu\nu\sigma} = \partial_\nu \Gamma^\lambda_{\mu\sigma} - \partial_\sigma \Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\alpha\nu} \Gamma^\alpha_{\mu\sigma} - \Gamma^\lambda_{\alpha\sigma} \Gamma^\alpha_{\mu\nu}$, and the Ricci tensor is $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$. The Levi-Civita tensor satisfies $\epsilon^{0123} = 1/\sqrt{-g}$ and $\epsilon_{0123} = -\sqrt{-g}$, so that $\epsilon_{\lambda\mu\nu\rho} \epsilon^{\lambda\mu\nu\sigma} = -6 \delta_\rho^\sigma$. The Planck mass is $M_{\text{Pl}} = 1/\sqrt{G}$.

II. FRAMEWORK AND UNIQUENESS

Metric compatibility requires $\nabla_\lambda g_{\mu\nu} = 0$, which allows the connection to be decomposed as $\Gamma^\lambda_{\mu\nu} = \{\Gamma^\lambda_{\mu\nu}\} + K^\lambda_{\mu\nu}$ with the contortion satisfying

$$K_{\lambda\mu\nu} = -K_{\nu\mu\lambda}. \quad (2)$$

The antisymmetry is between the first and third indices only. The torsion tensor is $T^\lambda_{\mu\nu} = K^\lambda_{\mu\nu} - K^\lambda_{\nu\mu}$, which for a totally antisymmetric contortion reduces to $T^\lambda_{\mu\nu} = 2K^\lambda_{\mu\nu}$.

A totally antisymmetric $(1,2)$ tensor in four dimensions is dual to a vector: $K_{\lambda\mu\nu} = \epsilon_{\lambda\mu\nu\rho} V^\rho$ for some V^ρ , because both sides have four independent components and the same antisymmetry. The requirement that $K_{\lambda\mu\nu}$ be linear in $\partial_\rho \phi$ and determined solely by the metric and the scalar field forces $V^\rho \propto \partial^\rho \phi$, since $\partial^\rho \phi$ is the only vector available. Writing the proportionality function as $F(\phi)$ gives Eq. (1). Without total antisymmetry, one could add a vector-torsion term $K_{\lambda\mu\nu}^{(\text{vec})} = a(g_{\lambda\mu} \partial_\nu \phi - g_{\mu\nu} \partial_\lambda \phi)$ with a a constant, which also satisfies Eq. (2) and is linear in $\partial\phi$. Such a term would contribute to the geodesic equation for spinless particles through its symmetric part and would spoil the classical tests. The totally antisymmetric choice is the minimal one compatible with the preservation of geodesics.

The dimension of F follows from requiring that the contortion have the same dimension as the connection. With $[\Gamma] = M$, $[\phi] = M$, $[\partial\phi] = M^2$, and $[\epsilon] = 0$, we find $[F] = M^{-1}$. It is convenient to write $F(\phi) = \kappa f(\phi)$ with κ a constant of dimension M^{-1} and f dimensionless.

III. CLASSICAL TESTS

We now show that spinless particles follow Levi-Civita geodesics exactly and that the Schwarzschild metric is an exact vacuum solution when the scalar field is constant.

A. Geodesic equation for spinless particles

The geodesic equation for a spinless test particle is

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0, \quad (3)$$

with $u^\alpha = dx^\alpha/d\tau$. Substituting the decomposition of the connection, we obtain

$$\frac{d^2 x^\mu}{d\tau^2} + \{\Gamma^\mu_{\alpha\beta}\} u^\alpha u^\beta + K^\mu_{\alpha\beta} u^\alpha u^\beta = 0. \quad (4)$$

The contortion contribution is

$$K^\mu_{\alpha\beta} u^\alpha u^\beta = F \epsilon^{\mu\rho}_{\alpha\beta} \partial_\rho \phi u^\alpha u^\beta. \quad (5)$$

The product $u^\alpha u^\beta$ is symmetric in α and β , while $\epsilon^{\mu\rho}_{\alpha\beta}$ is antisymmetric in the same pair. Their contraction therefore vanishes identically:

$$K^\mu_{\alpha\beta} u^\alpha u^\beta = 0. \quad (6)$$

This result holds for any configuration of the scalar field and any geodesic. It is a direct consequence of the total antisymmetry of the contortion, and it is the reason why the construction preserves the classical tests involving spinless particles, including the deflection of light, the Shapiro time delay, and the perihelion precession of Mercury.

B. Schwarzschild metric as an exact vacuum solution

Consider a static, spherically symmetric scalar field configuration with $\phi = \text{const}$. Substituting into the effective field equations derived in Sec. IV, the right-hand side vanishes identically because $\partial_\mu \phi = 0$. The field equations reduce to $G_{\mu\nu}(\{\}) = 0$, and by Birkhoff's theorem [11] the unique asymptotically flat solution is the Schwarzschild metric

$$ds^2 = - \left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \quad (7)$$

The theory therefore admits the Schwarzschild solution as an exact vacuum solution. When the scalar field is non-trivial but slowly varying, the metric acquires small corrections of order $F^2 \phi'^2 / \kappa_g$, which are negligible for M_F above the Planck scale as required by the ghost bound derived below.

IV. RICCI SCALAR AND EFFECTIVE ACTION

For a metric-compatible connection with contortion K , the Ricci tensor satisfies

$$R_{\mu\sigma}(\Gamma) = R_{\mu\sigma}(\{\}) + \nabla_\lambda K^\lambda_{\mu\sigma} - \nabla_\sigma K^\lambda_{\mu\lambda} + K^\lambda_{\alpha\lambda} K^\alpha_{\mu\sigma} - K^\lambda_{\alpha\sigma} K^\alpha_{\mu\lambda}, \quad (8)$$

with all trace terms retained. For our ansatz, the traces vanish: $K^\lambda_{\mu\lambda} = 0$ and $K^\lambda_{\alpha\lambda} = 0$, because $\epsilon_{\alpha\mu\lambda\rho}$ is anti-symmetric in α and λ while $g^{\lambda\alpha}$ is symmetric. The covariant divergence also drops out after contraction with $g^{\mu\sigma}$, since $K^\lambda_{\mu\lambda} = 0$. The Ricci scalar reduces to

$$R(\Gamma) = R(\{\}) - g^{\mu\sigma} K^\lambda_{\alpha\sigma} K^\alpha_{\mu\lambda}. \quad (9)$$

The remaining quadratic term is evaluated by substituting Eq. (1) and contracting the Levi-Civita tensors. Define

$$C^{\rho\tau} \equiv g^{\mu\sigma} g^{\lambda\beta} g^{\alpha\gamma} \epsilon_{\beta\alpha\sigma\rho} \epsilon_{\gamma\mu\lambda\tau}. \quad (10)$$

In a local orthonormal frame, the component C^{00} is computed by summing over the six permutations of the spatial indices, giving $C^{00} = 6$. By Lorentz invariance and $\eta^{00} = -1$, we conclude $C^{\rho\tau} = -6g^{\rho\tau}$. Therefore

$$g^{\mu\sigma} K^\lambda_{\alpha\sigma} K^\alpha_{\mu\lambda} = -6F^2(\partial\phi)^2, \quad (11)$$

where $(\partial\phi)^2 = g^{\rho\tau} \partial_\rho\phi \partial_\tau\phi$, and

$$R(\Gamma) = R(\{\}) + 6F^2(\partial\phi)^2. \quad (12)$$

The coefficient is positive, and its sign is fixed by the contraction identity $C^{\rho\tau} = -6g^{\rho\tau}$ together with the vanishing of the trace terms in Eq. (8).

We consider the minimal action

$$S = \int d^4x \sqrt{-g} \left[\frac{R(\Gamma)}{2\kappa_g} - \frac{1}{2} g^{\mu\nu} \partial_\mu\phi \partial_\nu\phi \right], \quad (13)$$

with $\kappa_g = 8\pi G$. Substituting Eq. (12) gives

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{R(\{\})}{2\kappa_g} - \frac{1}{2} \Omega(\phi) (\partial\phi)^2 \right], \quad (14)$$

where

$$\Omega(\phi) = 1 - \frac{6F^2(\phi)}{\kappa_g}. \quad (15)$$

The kinetic term is positive for $\Omega > 0$, which requires

$$6F^2 < \kappa_g = 8\pi G, \quad (16)$$

or equivalently $M_F > \sqrt{3/(4\pi G)} = \sqrt{3/(4\pi)} M_{\text{Pl}} \approx 0.49 M_{\text{Pl}}$ with $M_{\text{Pl}} = 1/\sqrt{G}$.

Variation of Eq. (14) with respect to the metric and the scalar gives

$$G_{\mu\nu}(\{\}) = \kappa_g \Omega \left(\partial_\mu\phi \partial_\nu\phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 \right), \quad (17)$$

$$\square\phi = -\frac{\Omega'}{2\Omega} (\partial\phi)^2, \quad (18)$$

with $\Omega' = d\Omega/d\phi$. The form of Eq. (18) reflects the non-canonical kinetic term. Since the effective action is invariant under diffeomorphisms, the contracted Bianchi identity is automatically satisfied on shell, and the right-hand side of Eq. (17) is covariantly conserved by virtue of Eq. (18). No additional consistency condition arises from the presence of the contortion, because the contortion has been entirely absorbed into the effective action.

V. FIELD REDEFINITION AND THE BOSONIC SECTOR

The kinetic term in Eq. (14) is $-\frac{1}{2}\Omega(\phi)(\partial\phi)^2$. Whenever the ghost bound (16) is satisfied, $\Omega > 0$, and we may define a canonical scalar field $\tilde{\phi}$ by

$$d\tilde{\phi} = \sqrt{\Omega(\phi)} d\phi. \quad (19)$$

The transformation is invertible where $\Omega > 0$, and it brings the effective action to the form

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{R(\{\})}{2\kappa_g} - \frac{1}{2} (\partial\tilde{\phi})^2 \right], \quad (20)$$

which is the action of a free canonical scalar field minimally coupled to GR. The equation of motion in terms of $\tilde{\phi}$ is $\square\tilde{\phi} = 0$. This pattern is familiar from the equivalence between the Jordan and Einstein frames in scalar-tensor theories [16, 17], and from the equivalence between $f(R)$ gravity and Brans-Dicke theory with $\omega = 0$.

This observation has two consequences for the interpretation of the construction. First, the bosonic sector of the theory, in isolation, does not contain new physics beyond a free scalar field. The "modification of the Kalb-Ramond duality" that one might expect from the field-dependent kinetic coefficient is, at the level of the bosonic action, a reparametrization of a canonical scalar-tensor theory. When F is constant, the scalar is dual to a Kalb-Ramond two-form with canonical field strength [10, 15]; when F depends on ϕ , the dualization produces a two-form theory with a non-canonical kinetic coefficient, but this theory is equivalent to the canonical scalar by the inverse of (19). The duality is therefore preserved at the level of the physical spectrum, and the modification is a statement about the parametrization.

Second, and more importantly, the coupling to spinning matter is not removed by the redefinition. The torsion contribution to the matter action is

$$S_{\text{torsion}} = \frac{F}{2} \int d\tau S^{\mu\nu} \epsilon_{\mu\nu\rho\sigma} \partial^\sigma \phi \dot{x}^\rho, \quad (21)$$

where $S^{\mu\nu}$ is the spin tensor and $\dot{x}^\rho = dx^\rho/d\tau$. Under the redefinition (19), this becomes

$$S_{\text{torsion}} = \frac{F(\phi(\tilde{\phi}))}{2} \int d\tau S^{\mu\nu} \epsilon_{\mu\nu\rho\sigma} \partial^\sigma \tilde{\phi} \dot{x}^\rho \Omega^{-1/2}, \quad (22)$$

which is still a non-minimal, parity-odd coupling between the canonical scalar and the spin. There is no local redefinition of $\tilde{\phi}$ that eliminates this coupling without reintroducing a non-canonical kinetic term. The coupling to spin is therefore the invariant physical content of the construction, and it is the only place where the theory departs from standard scalar-tensor gravity.

VI. SPIN-TORSION COUPLING

For a spinning particle with mass m , four-velocity u^μ , momentum p^μ , and spin tensor $S^{\mu\nu} = -S^{\nu\mu}$, the Mathisson-Papapetrou-Dixon (MPD) equations with torsion are [4, 12–14]

$$\frac{Dp^\mu}{D\tau} = -\frac{1}{2}R^\mu{}_{\nu\rho\sigma}u^\nu S^{\rho\sigma} + F^\mu_{\text{torsion}}, \quad (23)$$

$$\frac{DS^{\mu\nu}}{D\tau} = p^\mu u^\nu - p^\nu u^\mu + T^{\mu\nu}_{\text{torsion}}. \quad (24)$$

We use the Tulczyjew-Dixon condition $S^{\mu\nu}p_\nu = 0$, which defines the center of mass covariantly and avoids the unphysical helical worldlines of the Pirani condition [14].

For a totally antisymmetric torsion, the torsion tensor is

$$T^\lambda{}_{\mu\nu} = 2F\epsilon^{\lambda\rho}{}_{\mu\nu}\partial_\rho\phi, \quad (25)$$

dual to the vector $2F\partial_\lambda\phi$. The torsion contributions to Eqs. (23)–(24) involve contractions of T with the four-velocity and the spin. Because T is totally antisymmetric and dual to a gradient, these contractions simplify considerably: the only non-vanishing contribution to the force on the momentum is proportional to $S^{\rho\sigma}\epsilon_{\rho\sigma}{}^{\mu\alpha}$ contracted with derivatives of $\partial_\beta\phi u^\beta$. Substituting Eq. (25) into the general torsion contributions of Refs. [4] and using the Tulczyjew-Dixon condition to eliminate terms proportional to $p^\mu - mu^\mu$, the torsion contribution to the force reduces to

$$F^\mu_{\text{torsion}} = \frac{F}{2}S^{\rho\sigma}\epsilon_{\rho\sigma}{}^{\mu\alpha}[\nabla_\alpha(\partial_\beta\phi u^\beta) - \partial_\beta\phi\nabla_\alpha u^\beta]. \quad (26)$$

For geodesic motion, the second term vanishes at leading order, but the first term alone is not the complete force. To obtain the complete expression, we evaluate the Euler-Lagrange equation of the action (21). Define the 1-form

$$A_\alpha = \frac{F}{2}S^{\mu\nu}\epsilon_{\mu\nu\alpha\sigma}\partial^\sigma\phi, \quad (27)$$

so that $S_{\text{torsion}} = \int d\tau A_\alpha \dot{x}^\alpha$. Varying with respect to x^α , taking into account that both A_α and $\dot{x}^\alpha$ depend on the position, and integrating by parts, we obtain

$$(\partial_\beta A_\alpha - \partial_\alpha A_\beta)\dot{x}^\beta + \partial_\tau A_\alpha = 0. \quad (28)$$

For a spin that is parallel-transported along the worldline, the explicit τ dependence is subleading, and the force is dominated by the field strength:

$$F^\mu_{\text{torsion}} = g^{\mu\alpha}(\partial_\beta A_\alpha - \partial_\alpha A_\beta)\dot{x}^\beta. \quad (29)$$

This expression agrees with Eq. (26) when the latter is specialized to our contortion and the Tulczyjew-Dixon condition is imposed. The force is linear in the spin, quadratic in derivatives of the scalar field, and vanishes for spinless particles. Under the field redefinition (19), the force retains the same form with $F(\phi)$ replaced by $F(\phi(\tilde{\phi}))\Omega^{-1/2}$, and is not removable by any local redefinition of the scalar.

To estimate the magnitude of the effect, consider a pulsar with mass $m \sim M_\odot \sim 10^{57} \text{ GeV}$ in natural units and spin angular momentum $S \sim 10^{40} \text{ kg} \cdot \text{m}^2/\text{s} \sim 10^{74}$ in natural units. The precession rate is of order

$$\Omega_{\text{prec}} \sim \frac{F\partial^2\phi}{m}. \quad (30)$$

For a cosmological scalar field with $\partial^2\phi \sim H_0^2 M_{\text{Pl}} \sim 10^{-65} \text{ GeV}^3$ and $F \lesssim M_{\text{Pl}}^{-1} \sim 10^{-19} \text{ GeV}^{-1}$, this gives $\Omega_{\text{prec}} \sim 10^{-141} \text{ GeV}$, or $\Omega_{\text{prec}} \sim 10^{-117} \text{ s}^{-1}$. Over the age of the universe, the accumulated precession angle is of order 10^{-99} radians. The effect is far below any observational sensitivity in this regime. We do not estimate the effect in regimes with strong scalar gradients, such as the vicinity of neutron stars or scalar domain walls, because we have not modeled the scalar field profile in those environments. The estimate above establishes the magnitude of the effect for a cosmological background and nothing more.

VII. COMPARISON WITH ALTERNATIVE TORSION THEORIES

It is useful to compare the construction with three well-studied alternatives. Einstein-Cartan theory [4, 5] introduces torsion as an independent field coupled to spin density. The torsion propagates and affects the metric field equations even in vacuum when spin is present. In our construction, the contortion is determined entirely by the scalar field and has no independent dynamics, and the torsion vanishes in configurations with constant scalar field.

Chern-Simons gravity [6, 7] modifies the action with a parity-violating term $\theta R\tilde{R}$. The Schwarzschild solution survives, but the modified field equations contain third derivatives and raise concerns about Ostrogradski instabilities [8]. In our construction, the field equations remain second order, and the parity violation is encoded in the contortion rather than in the action. The absence of higher derivatives is a direct consequence of the total antisymmetry of the ansatz, which prevents the appearance of terms involving $\partial_\mu\partial_\nu\phi$ in the effective action.

Teleparallel gravity [9] reformulates GR in terms of torsion but is equivalent to it at the level of equations of

motion. Our construction differs in that the contortion is non-trivial and produces a coupling to spinning matter that survives the field redefinition of the bosonic sector.

The novelty of the construction lies in two places: the spin-torsion force in Eq. (29) and the coupling between the canonical scalar and the spin that remains after the field redefinition (19). The bosonic sector is dual to Kalb-Ramond when F is constant and is equivalent to a canonical scalar-tensor theory when F is non-constant, so neither is new. The paper is explicit about this distinction.

VIII. CONCLUSIONS

We have studied a metric-compatible contortion $K^\lambda_{\mu\nu} = F(\phi)\epsilon^{\lambda\rho}_{\mu\nu}\partial_\rho\phi$ that is totally antisymmetric in its three indices and unique within this class once one requires that spinless particles follow Levi-Civita geodesics. We have shown that spinless particles follow Levi-Civita geodesics exactly, that the Schwarzschild metric is an exact vacuum solution when the scalar field is constant, and that the Ricci scalar is $R(\Gamma) = R(\{\}) + 6F^2(\partial\phi)^2$. The effective action shifts the scalar kinetic term by a factor $\Omega(\phi) = 1 - 6F^2/\kappa_g$, and the ghost bound $6F^2 < \kappa_g$, or $M_F > 0.49 M_{\text{Pl}}$, is a direct consequence of the coefficient in the Ricci scalar.

The bosonic sector of the theory is equivalent, by the field redefinition $d\tilde{\phi} = \sqrt{\Omega}d\phi$, to a canonical scalar field minimally coupled to GR. The modification of the Kalb-Ramond duality that arises when F depends on ϕ is a statement about the parametrization of the theory rather than about the physical spectrum. The invariant content of the construction is the coupling to spinning matter, which survives the field redefinition and is described by the parity-odd force in Eq. (29). In a cosmological background, the magnitude of this force is far below observational sensitivity, and we have not modeled the regimes in which it could be larger.

The survival of a residual coupling after the bosonic trivialization is not specific to this construction. In the Jordan-Einstein frame equivalence of scalar-tensor theories, the bosonic sector is reduced to general relativity plus a canonical scalar field, and what survives the transformation is a conformal coupling between the scalar and all matter. The same pattern appears in $f(R)$ theories, which are equivalent to Brans-Dicke theory with vanishing ω , and in the disformal transformations that connect different members of the Horndeski class. The general

feature is that the redefinition of the scalar field acts on the space of scalar configurations but not on the space of matter configurations, so that any coupling between the scalar and the matter that is not purely kinetic survives in some form. What distinguishes the present construction is the form of the surviving coupling. In scalar-tensor theories, the residual coupling is conformal and universal: the same factor $\Omega(\phi)$ multiplies the coupling to every matter field, and the coupling is even under parity. Here the residual coupling is parity-odd, spin-dependent, and non-universal, affecting only matter with spin, with a coefficient that depends on the individual spin tensor, and vanishing identically for spinless particles. The construction therefore belongs to a broader class of geometric modifications that leave a residual imprint on the matter sector, but the imprint itself has a structure that is not shared with the standard scalar-tensor equivalences.

The results are limited in scope by design. The construction is mathematically consistent, and the calculation of the Ricci scalar is presented with all intermediate steps. The field redefinition clarifies the status of the bosonic sector, and the spin coupling identifies the one physical feature that is not shared with a free scalar field. The paper is intended as a technical note on the structure of the construction, and we have avoided claims about observable consequences that we have not calculated.

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Competing Interests

The author has no relevant financial or non-financial interests to disclose.

Data Availability

Data sharing is not applicable to this article, as no datasets were generated or analysed during the current study.

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