

Note to Analytical Derivation of Viswanath's Constant.

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Abstract: *In the article, we examine the control functions that enable the analytical derivation of Viswanath's constant.*

Derivation of Viswanath's Constant

Deriving Viswanath's constant is extremely difficult. Divakar Viswanath was the first to do so in 1999 [1]. In our recent paper, we derived a closed-form approximation of Viswanath's constant using a control function $g(x)$ [2]. We quote:

„There is also a physical interpretation of the function $g(x)$ and its relationship to the Viswanath's constant. The function $g(x)$ can be considered as a wave function very well normalized in the interval $(-1, 1)$. The point $x=0$ represents the present, the interval $(-1, 0)$ the causal past, and the interval $(0, 0.5)$ the causal future. Although chance plays a large role in RFS, causality is always present, due to its predictable rate of its growth expressed by the Viswanath constant. It is actually the length of the curve bounding the causal region of the function $g(x)$. Let us consider a system where two options, + and – are decisive. This system has Shannon entropy $\ln 2$. If we find a curve that in the interval (a, b) bounds the area $\ln 2$ and at the same time this curve is normalized to 1, then we claim that the length of this curve in the interval (a,b) exactly approximates Viswanath's constant“ It holds:

$$g(x) = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad (1)$$

$$\int_{-1}^0 g(x) dx = \int_0^{0.5} g(x) dx = \ln 2 \quad (2)$$

$$\int_{-1}^0 g^2(x) dx = \int_0^{0.5} g^2(x) dx = 1 \quad (3)$$

$$\int_{-1}^0 \sqrt{1 + \left(\frac{dg(x)}{dx}\right)^2} dx = \int_0^{0.5} \sqrt{1 + \left(\frac{dg(x)}{dx}\right)^2} dx = 1.13209..... \quad (4)$$

Relation (4) represents an approximation of Viswanath's constant.

We have found a function $f(x)$ that also partially satisfies conditions (2), (3), and (4) on the interval $(-0.5, 0)$. The following holds:

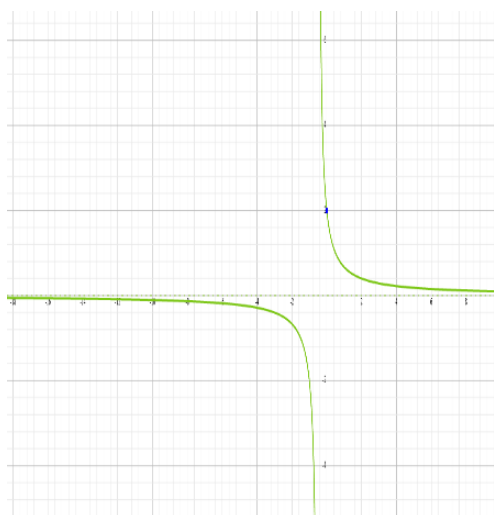
$$f(x) = \frac{1}{x+1} \quad (5)$$



Picture 1: *Graph of the function $f(x)$*

And furthermore, the function $c(x)$, which satisfies the stated conditions on the interval $(0, 0.5)$. The following holds:

$$c(x) = \frac{2}{1+2x} \quad (6)$$



Picture 2: *Graph of the function $c(x)$*

In conclusion, we can state:

$$\int_{-0.5}^0 f_{(x)} dx = \int_0^{0.5} c_{(x)} dx = \ln 2 \quad (7)$$

$$\int_{-0.5}^0 f_{(x)}^2 dx = \int_0^{0.5} c_{(x)}^2 dx = 1 \quad (8)$$

$$V = \int_{-0.5}^0 \sqrt{1 + \left(\frac{df(x)}{dx}\right)^2} dx = \int_0^{0.5} \sqrt{1 + \left(\frac{dc(x)}{dx}\right)^2} dx = 1.13209 \dots \quad (9)$$

Conclusions:

Although we found other functions approximating Viswanath's constant, there is no causal relationship between the past and the future in their case. Therefore, the best and simplest option remains the use of the control function $g(x)$.

References:

[1]: Viswanath, D. (1999). *Mathematics of Computation*. „Random Fibonacci sequences and number 1,13198824...” Volume 69, Number 231, Pages 1131–1155 S 0025-5718(99)01145-X

[2]: Súkeník, M. and Súkeníková, M. (2026) Approximation of Viswanath's Constant in Closed Form. *Journal of High Energy Physics, Gravitation and Cosmology*, **12**, 857-865.

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