

A Critique Of John Bell's 1964 Article On The Einstein Podolsky Rosen Paradox

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Abstract

This article identifies a series of foundational mathematical and logical inconsistencies within John Bell's 1964 formulation [1] and demonstrates that the core discrepancy in Bell's framework arises from an implicit and unjustified data-mapping assumption which treats the discrete, binary-valued random functions of [1, equation (2)] as directly equivalent to the continuous, real-valued integrands of [1, equation (3)]. This non-linear transformation incurs a severe loss of contextual information that is mathematically unaccounted for in Bell's original derivation. By failing to recognize this measure-theoretic mismatch the subsequent deductions, including the assertion that a remote measurement setting must dynamically influence a distant apparatus, are rendered logically invalid. Consequently, the attribution of non-locality to Einstein-Podolsky-Rosen-Bohm (EPRB) systems is shown to be an unfounded artifact of this misapplied probability space rather than an empirical property of physical nature.

1 Introduction

This article identifies a number of incorrect and misleading statements in [1].

The key erroneous statement in [1] is that the expectation value of a binary valued function [1, equation (2)] should equal the expectation value of a related real number valued function of [1, equation (3)].

This erroneous statement leads the content of [1] to other the illogical conclusions including the unfounded and illogical conclusion that non-locality is involved in the interpretation of Stern-Gerlach, and by implication, EPRB¹ experiments.

2 Critique

2.1 Item 1

This statement in [1, Section II. Formulation] is misleading:

“Measurements can be made, say by Stern-Gerlach magnets, on selected components of the Spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$ ”

Stern-Gerlach devices measure the polarity of spins in a local reference frame rather than actual real number intrinsic spin values. Precise measurement of an intrinsic magnetic moment (spin) requires a sensitive magnetometer.

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¹Einstein-Podolsky-Rosen-Bohm

2.2 Item 2

The statement from [1, Section II. Formulation] is incorrect that [1, equation (2)]:

$$P(\vec{a}, \vec{b}) = \int d\lambda \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) \quad (1)$$

should equal the quantum mechanical expectation value [1, equation (3)]:

$$\langle \vec{\sigma}_1 \cdot \vec{a} \vec{\sigma}_2 \cdot \vec{b} \rangle = -\vec{a} \cdot \vec{b}. \quad (2)$$

This statement indicates that the author did not have an adequate understanding of the meaning and interpretation of the statistical term “expectation value”.

The phrase, “the quantum mechanical expectation value” is an incomplete description as it does not state which quantum mechanical expectation value is being referred to. The complete statement is:

“the quantum mechanical expectation value of the product of the two observables $\vec{\sigma}_1 \cdot \vec{a}$ and $\vec{\sigma}_2 \cdot \vec{b}$ ”.

$\rho(\lambda)$ in equation (1) represents the probability distribution frequency of the random variable λ over its domain.

Equation (1) describes the expectation value $P(\vec{a}, \vec{b})$ of the product of the values of two binary functions, $A(\vec{a}, \lambda)$ and $B(\vec{b}, \lambda)$, over a binary measurement domain whereas equation (2) describes the expectation value $\langle \vec{\sigma}_1 \cdot \vec{a} \vec{\sigma}_2 \cdot \vec{b} \rangle$ of the product of two real numbers $\vec{\sigma}_1 \cdot \vec{a}$ and $\vec{\sigma}_2 \cdot \vec{b}$ over a measurement domain of vectors of real-numbers ($\mathbb{R}$).

In fact:

$$P(\vec{a}, \vec{b}) = \frac{2}{\pi} |\vec{a} - \vec{b}| - 1 \quad (3)$$

which, in general, is not equal to $-\vec{a} \cdot \vec{b}$. In fact, Bell obtains equation (3) at [1, equation (10)].

The difference between the two expectation values (1) and (2) can be appreciated more clearly if the functions $A(\vec{a}, \lambda)$ and $B(\vec{b}, \lambda)$ are considered as each being composed of two parts; a first part in which the random variable λ is rotated into alignment with a local reference frame and a second part in which the result of the first part is transformed by a sign function into a binary value.

The expectation value of the first part is then given by equation (2) which is then transformed by the sign function into a binary value the expectation value of which is given by equation (1).

The difference between these expectation values is illustrated by Figure 1.

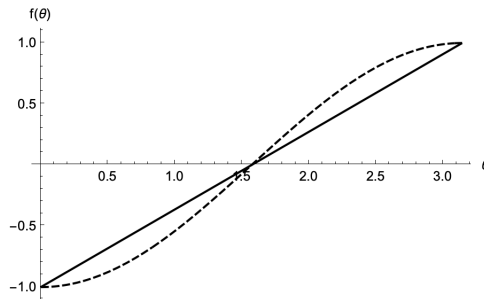


Figure 1: Comparison of the results of equations (1) (solid line) and (2) (dashed line)

2.3 Item 3

The symbols $\vec{\sigma}_1$ and $\vec{\sigma}_2$ of equation (2) are unstated redefinitions of the symbols previously defined as spins. $\vec{\sigma}_1$ and $\vec{\sigma}_2$ in equation (2) represent the purely mathematical vector of three Pauli matrices operating locally on the vectors $\vec{a}$ and $\vec{b}$ to form the observables $\vec{\sigma}_1 \cdot \vec{a}$ and $\vec{\sigma}_2 \cdot \vec{b}$. These observables do not operate on each other as implied by equation (2) but operate locally on different random variables; for example, the random variables λ_1 and $\lambda_2 = -\lambda_1$.

$\langle \vec{\sigma}_1 \cdot \vec{a} \vec{\sigma}_2 \cdot \vec{b} \rangle$ might better be defined as $\langle \vec{\sigma} \cdot \vec{a} \otimes \vec{\sigma} \cdot \vec{b} \rangle$, $\vec{\sigma}$ representing the vector of the three Pauli matrices with $\otimes$ indicating composition rather than multiplication.

2.4 Item 4

In the derivation of [1, equation (15)], individual components of the operator $A(b, \lambda)A(b, \lambda)$ transform the expression $\langle A(a, \lambda)B(b, \lambda) \rangle - \langle A(a, \lambda)B(c, \lambda) \rangle$ into the expression $\langle A(a, \lambda)B(b, \lambda) \rangle - \langle A(b, \lambda)B(c, \lambda) \rangle$.

Subsequent analysis of the expression $\langle A(a, \lambda)B(b, \lambda) \rangle - \langle A(b, \lambda)B(c, \lambda) \rangle$ yields the inequality:

$$1 + \langle A(a, \lambda)B(c, \lambda) \rangle \geq |\langle A(a, \lambda)B(b, \lambda) \rangle - \langle A(a, \lambda)B(c, \lambda) \rangle| \quad (4)$$

which is not the same as Bell's inequality [1, equation (15)]:

$$1 + \langle A(b, \lambda)B(c, \lambda) \rangle \geq |\langle A(a, \lambda)B(b, \lambda) \rangle - \langle A(a, \lambda)B(c, \lambda) \rangle| \quad (5)$$

2.5 Item 5

The following statement appears in [1, Section IV. Contradiction]:

“Suppose that for all $\vec{a}$ and $\vec{b}$ the difference” between $\overline{P(\vec{a}, \vec{b})}$ and $-\overline{\vec{a} \cdot \vec{b}}$ “is bounded by ϵ : [1, equation (16)]

$$\left| \overline{P(\vec{a}, \vec{b})} + \overline{\vec{a} \cdot \vec{b}} \right| \leq \epsilon \quad (6)$$

where the bar indicates independent averaging of $P(\vec{a}', \vec{b}')$ and $-\vec{a}' \cdot \vec{b}'$ over vectors $\vec{a}'$ and $\vec{b}'$ within specified small angles of $\vec{a}$ and $\vec{b}$.”

“Then it will be shown that ϵ cannot be made arbitrarily small.”

But this difference is to be expected as $P(\vec{a}', \vec{b}')$ and $-\vec{a}' \cdot \vec{b}'$ are different expectation values over different measurement domains and, consequently, should be expected, in general, to have different values .

So showing that “ ϵ cannot be made arbitrarily small” proves that the statement referred to in Section 2.2 that $P(\vec{a}, \vec{b})$ and $-\vec{a} \cdot \vec{b}$ should be equal is indeed false.

Similarly the statement at the end of [1, section IV. Contradiction] that “the quantum mechanical expectation value cannot be represented, either accurately or arbitrarily closely, in the form (2)” is a truism.

2.6 Item 6

The following statement occurs at the end of [1, End of Section III. Illustration]:

“There is no difficulty in reproducing the quantum mechanical correlation [1, equation (3)] if the results A and B in [1, equation (2)] are allowed to depend on b and a respectively as well as on a and b.”

This may be mathematically true. However, without justification, it defies the axiom of locality of quantum field theory that governs the propagation of changes in proper values and, hence, is unacceptable.

Therefore, while this parameter cross-dependence is algebraically trivial, it is physically and procedurally unacceptable within the context of an Einstein-Podolsky-Rosen-Bohm (EPRB) framework for two distinct reasons:

1. Violation of the Locality Axiom:

Allowing the local binary function $A(\lambda, a)$ to depend on the distant setting b (and vice versa for $B(\lambda, \vec{b})$) explicitly violates the quantum field theory foundational axiom of local parameter independence and introduces an ad-hoc, non-local dependence without any physical or measure-theoretic justification.

2. Irrelevance to Empirical Protocols:

Real-world EPRB experiments are designed specifically to implement independent, isolated local functions $A(\lambda, \vec{a})$ and $B(\lambda, \vec{b})$. The joint expectation value of these isolated laboratory operations is explicitly defined by [1, equation (2)]. Consequently, introducing cross-dependent parameters creates an entirely separate mathematical system that has no logical involvement in the interpretation of discrete, binary experimental outcomes.

The apparent discrepancy between [1, equation (2)] and [1, equation (3)] is not a symptom of physical non-locality but rather a direct consequence of the information loss incurred when mapping continuous statistical expectation formulae to binary-valued random number spaces.

2.7 Item 7

The following definitive deduction regarding local hidden variable frameworks is stated in [1, Section VI. Conclusion]:

“In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote.”

This sweeping conclusion relies entirely on the validity of the cross-dependence assumption addressed in Section 2.6.

Because the cross-dependence has been shown to be structurally irrelevant to establishing the true joint expectation values of independent local functions $A(\lambda, \vec{a})$ and $B(\lambda, \vec{b})$ the deduction built upon it is logically invalid.

Furthermore, while [1, Section VI. Conclusion] explicitly restricts this assertion to theories "in which parameters are added to quantum mechanics" a significant portion of the physics community has historically accepted this conclusion without rigorous justification as a generalized property of nature itself.

By failing to recognize that the original discrepancy stems from a measure-theoretic mismatch between continuous expectation formulas and binary-valued random spaces the mainstream interpretation mistakes a localized data-transformation artifact for a profound physical non-locality.

3 Conclusion

The foundational conclusion presented in [1, Section VI. Conclusion] asserting that the setting of a remote measuring device must dynamically influence a distant instrument is structurally unfounded as it relies entirely on a fundamental category error, the mathematically unjustified assumption that the binary joint expectation

value defined by equation (1) must equate directly to the continuous, real-valued expectation integral of equation (2).

In mathematical reality, equation (1) represents a distinct formal extension of equation (2) specifically structured to accommodate the probability bounds of discrete, binary-valued functions.

Because this non-linear data mapping incurs a severe, unmeasured loss of contextual information, the resulting discrepancy is an artifact of the mathematical framework itself rather than a physical reality. Consequently, because Bell's primary deduction regarding remote parameters is invalid, all subsequent theoretical frameworks and theorems derived from this boundary mismatch are similarly invalidated under the strict discipline of the Scientific Method.

References

- [1] J. S. Bell. On the einstein-podolsky-rosen paradox. *Physics*, 1:195–200, 1964.