# Gauss's flux theorem in differential form has no definite physical significance

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**Abstract:** In classical electromagnetism, Gauss's flux theorem in integral form states that the electric flux through a closed surface is proportional to the charge enclosed by that surface. Recent research points out that Gauss's flux theorem in integral form applies to static electric fields but not to time-varying electric fields. This paper reveals that when Gauss's flux, characterized by three-dimensional space, is compressed to a single point, the divergence of a field no longer retains its original meaning of the flux. Based on the divergence of the electric field at a point in space, Gauss's flux theorem in differential form contradicts Coulomb's law and violates the superposition principle of the electric fields. Therefore, the differential and integral forms of Gauss's flux theorem are not equivalent. In conclusion, Gauss's flux theorem in differential form does not hold in both static and time-varying electric fields, and has no definite mathematical and physical significance. Theoretical physics and mathematics will undergo a revolutionary scientific transformation.

**Keywords**: Coulomb's law, Gauss's flux theorem in integral form, Gauss's flux theorem in differential form, flux-limit divergence, nabla-operator divergence.

## 1. Introduction

Maxwell's equations were the greatest scientific achievement of physics in the 19th. Maxwell's equations describe the relationship among charges, currents, electric fields, and magnetic fields, proposing the theory of electromagnetic waves and predicting that light is an electromagnetic wave century [1-6]. Below are Maxwell's equations in integral form.By

$$\oiint_s E \cdot ds = \frac{Q}{\varepsilon_0} \qquad (1\text{-}1A)$$

$$\oiint_s B \cdot ds = 0 \qquad (1\text{-}2A)$$

$$\oint_L E \cdot d\ell = -\frac{d\Phi_B}{dt} \qquad (1\text{-}3A)$$

$$\oint_L B \cdot d\ell = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad (1\text{-}4A)$$

introducing the divergence, curl, and the nabla-operator (∇ operator), the integral form of Maxwell's equations is converted into differential form. Below are Maxwell's equations in differential form.

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \qquad (1\text{-}1B)$$

$$\nabla \cdot B = 0 \qquad (1\text{-}2B)$$

$$\nabla \times E = \frac{\partial B}{\partial t} \qquad (1\text{-}3B)$$

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \qquad (1\text{-}4B)$$

Equations (1-1A) and (1-1B) represent Gauss's flux theorem in integral form and differential form, respectively. Equations (1-2A) and (1-2B) represent the magnetic flux theorem in integral form and differential form, respectively.

Coulomb's law describes the electric force between two point charges in a vacuum. For two point charges $q_1$ and $q_2$ separated in space, the electric force exerted from $q_1$ on $q_2$ can be expressed as:

$$\boldsymbol{F} = \frac{q_1 q_2}{4\pi\,\varepsilon_0\, r^2}\boldsymbol{r} \qquad (1-5)$$

where r is the distance between $q_1$ and $q_2$, **r** is the unit vector directed from $q_1$ to $q_2$, and $\varepsilon_0$ is the vacuum permittivity.

The electric field intensity ***E*** is defined as the electric force per unit test charge. For a point charge Q at distance r, the electric field intensity is:

$$\boldsymbol{E} = \frac{\mathrm{Q}}{4\pi\,\varepsilon_0\, r^2}\boldsymbol{r} \qquad (1-6)$$

Gauss's flux theorem follows directly from Coulomb's law of electrostatics. As shown in Fig. 1.1, for a surface element d**S** in three-dimensional space, the electric flux $d\Phi_E$ equals the product of the electric field intensity $\boldsymbol{E}\,(x, y, z)$ and the element's effective area d**S**.

$$d\Phi_E = \boldsymbol{E}\,(\mathrm{x, y, z}) \cdot d\mathbf{S} = \mathrm{E\, dS\, Cos}\,\theta$$

where θ is the angle between the electric field $\boldsymbol{E}\,(x, y, z)$ and the outward normal to the surface element d**S**.

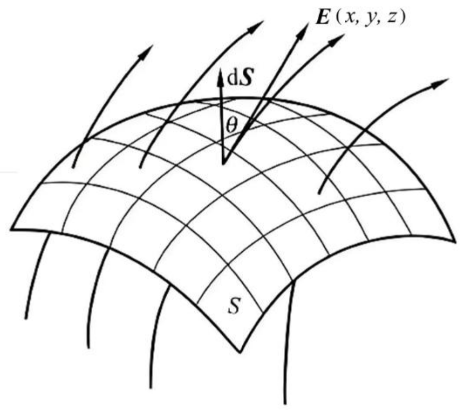


Fig. 1.1 Electric flux through a surface

In three-dimensional space, the electric flux $\Phi_E$ through a general surface S is given by the surface integral:

$$\Phi_E = \oiint_S \boldsymbol{E}\,(x, y, z) \cdot d\mathbf{S} \qquad (1-7)$$

Gauss's flux theorem, derived from Coulomb's law of electrostatics, states that the electric flux through a closed surface S is proportional to the net charge Q enclosed by that surface. Mathematically, this is expressed as:

$$\oiint_S \boldsymbol{E}(x, y, z) \cdot \mathrm{d}\mathbf{S} = \frac{Q}{\varepsilon_0} \qquad (1-8A)$$

$$\nabla \cdot \boldsymbol{E}(x, y, z) = \frac{\rho(x, y, z)}{\varepsilon_0} \qquad (1-8B)$$

Equation (1-8A) is Gauss's flux theorem in integral form, which is Gauss's flux theorem in classical electromagnetism. Equation (1-8B) is Gauss's flux theorem in differential form based on the nabla-operator divergence, where $\rho(x, y, z)$ denotes the charge density.

Before 1850, experiments and observations had led to the discovery of Coulomb's law, Gauss's flux theorem, Ampere's force theorem, Biot-Savart theorem, Ampere's circuital theorem, and Faraday's law of electromagnetic induction. Classical electromagnetism had been basically established. Based on divergence, curl, and the nabla-operator, Maxwell's equations are an integration of partial differential equations and classical electromagnetism, they are not part of classical electromagnetism.

It is a common misconception in physics to treat Maxwell's equations as part of classical electromagnetism. In engineering practice, no direct applications of Maxwell's equations can be found. The great success of classical electromagnetism in engineering applications has nothing to do with Maxwell's equations. In contrast, Maxwell's equations have hindered the development of classical electromagnetism; for example, Maxwell's equations impede the extension of Coulomb's law into the regime of electrodynamics [7].

In classical electromagnetism, Gauss's flux theorem in integral form is originally derived from Coulomb's law of electrostatics. However, Maxwell's equations incorrectly extend Gauss's flux theorem to apply to time-varying electric fields. In a static electric field, the electric field entering a closed surface is equal to the field leaving it. By contrast, in a time-varying electric field, the electric field entering a closed surface is not equal to the field leaving it.

Recent research points out that Gauss's flux theorem in integral form applies to static electric fields but not to time-varying electric fields [8]. Hertz's verification experiment in 1887, based on high-voltage discharge sparks, did not prove the existence of "electromagnetic waves"; but rather proved that wireless communication was achieved by independent "electric field waves" [9]. In 2024, according to quantitative measurements, an experiment revealed that in the radiation fields of an antenna, the electric field energy density is 137.6 times greater than the magnetic field energy density [10]; that is, the experimental result differs from the predicted value of the "electric field waves" theory by a factor of 137.6. This is the first verification experiment of "electromagnetic waves" based on the requirements of quantitative experiments in the history of science and technology. In charged moving media, Maxwell's equations are inconsistent with engineering practice, and an extension of conventional Maxwell equations has been proposed to charged moving media [11] [12]. Maxwell's equations and theoretical physics are facing serious challenges.

## 2. Definition of divergence and nabla-operator (∇ operator) [13] [14]

### Definition of the divergence of a vector *F*(*x, y, z*)

Let a closed surface S enclose a three-dimensional solid volume ΔV. As ΔV → 0, the divergence of the vector ***F***(*x, y, z*) is defined as the limit of the ratio of the flux of ***F***(*x, y, z*) through S to the volume ΔV:

$$\operatorname{div} \boldsymbol{F}(x, y, z) = \lim_{\Delta V \to 0} \frac{\oiint_S \boldsymbol{F}(x, y, z) \bullet d\boldsymbol{S}}{\Delta V} \qquad (2-1)$$

Equation (2-1) defines the vector divergence in terms of the limit of the flux, which is referred to as the **flux-limit divergence**.

### Definition of the nabla-operator (∇ operator)

In the rectangular coordinate system, the nabla-operator is defined as follows:

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k} \qquad (2-2)$$

where ***i, j,*** and ***k*** are the unit vectors in the rectangular coordinate system.

With nabla-operator, the divergence of the vector ***F***(*x, y, z*) can be expressed as follows:

$$\operatorname{div} \boldsymbol{F}(x, y, z) = \nabla \bullet \boldsymbol{F}(x, y, z) \qquad (2-3)$$

Equation (2-3) defines the vector divergence in terms of the partial differential nabla-operator, which is referred to as the **nabla-operator divergence**.

According to the definitions of divergence and nabla-operator, a three-dimensional vector

$$\boldsymbol{F}(x, y, z) = \boldsymbol{Fx}\,\boldsymbol{i} + \boldsymbol{Fy}\,\boldsymbol{j} + \boldsymbol{Fz}\,\boldsymbol{k}$$

Its divergence can be expressed as follows:

$$\nabla \bullet \boldsymbol{F}(x, y, z) = \frac{\partial \boldsymbol{Fx}}{\partial x} + \frac{\partial \boldsymbol{Fy}}{\partial y} + \frac{\partial \boldsymbol{Fz}}{\partial z}$$

$$\nabla \bullet \boldsymbol{F}(x, y, z) = \lim_{\Delta V \to 0} \frac{\oiint_S \boldsymbol{F}(x, y, z) \bullet d\boldsymbol{S}}{\Delta V} \qquad (2-4)$$

In the two definitions of divergence presented in Equation (2-4), the left-hand side denotes the **nabla-operator divergence**, and the right-hand side denotes the **flux-limit divergence**. The

**flux-limit divergence** provides the physical definition of divergence, whereas the **nabla-operator divergence** serves as a mathematical tool for calculation and derivation [15]. In theoretical physics, for example, Maxwell's equations and electrodynamics, the **nabla-operator divergence** is generally employed.

In classical electromagnetism, Gauss flux is the surface integral of the electric field over a three-dimensional closed surface, and it has a characteristic of three-dimensional space. In Equation (2 - 4), the **nabla-operator divergence** is the partial derivative of a vector at a single point in space and does not possess a characteristic of three-dimensional space.

## 3. Gauss's flux theorem in differential form has no definite physical significance

Gauss's flux theorem in integral form applies to static electric fields but not to time-varying electric fields. Based on the divergence of the electric fields at a point, Gauss's flux theorem in differential form does not hold in both static and time-varying electric fields, and has no definite mathematical and physical meanings. Below, we demonstrate these claims with examples based on Coulomb's law of electrostatics.

Consider a static point charge q in a vacuum. Choose an arbitrary point P at distance $x$ from q. Take the point charge q as the origin and the line from q to P as the $x$-axis. The electric field at any point on the $x$-axis has only an x-component, so the field $\boldsymbol{E}(x)$ can be treated as a one-dimensional static electric field, as shown in Figure 3.1.

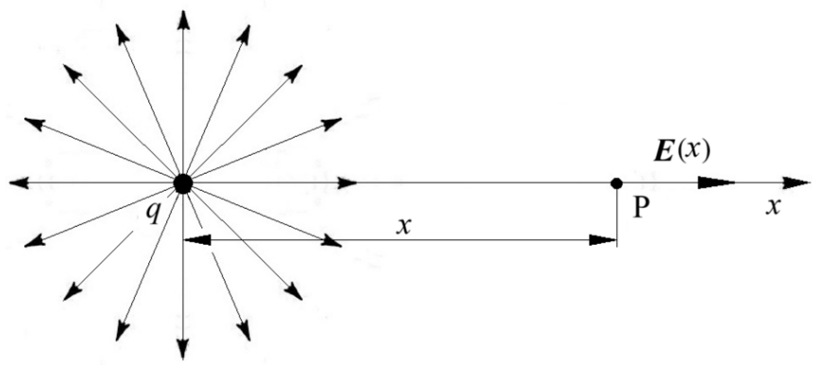


Figure 3.1 Nabla-operator divergence of the field of a point charge

In a vacuum, the charge density ρ equals zero. Applying the three-dimensional Gauss's flux theorem in differential form (Equation (1-8B)) and restricting it to the $x$-axis, the one-dimensional Gauss's flux theorem in differential form is given as follows:

$$\frac{\mathrm{d}\boldsymbol{E}(x)}{\mathrm{d}x} = 0 \qquad (3-1)$$

According to Coulomb's law, the electric field intensity at point P is:

According to the one-dimensional Gauss's flux theorem in differential form (Equation (3-1)),

$$\boldsymbol{E}(x) = \frac{\mathrm{q}}{4\pi\,\varepsilon_0\,x^2}$$

the nabla-operator divergence of the electric field **E**(x) is:

$$\frac{\mathrm{d}\boldsymbol{E}(x)}{\mathrm{d}x} = \frac{\mathrm{q}}{4\pi\,\varepsilon_0}\,\mathrm{d}(\frac{1}{x^2})/\mathrm{d}x = -\frac{\mathrm{q}}{2\pi\,\varepsilon_0\,x^3} \qquad (3-2)$$

Equation (3-2) implies the left-hand side of Equation (3-1) is nonzero while its right-hand side is identically zero, so Equation (3-1) does not hold. Consequently, **when the charge density ρ = 0, Gauss's flux theorem in differential form, which relies on the nabla-operator divergence, does not hold for a static electric field.** By a similar argument, when the charge density ρ = 0, Gauss's flux theorem in differential form also fails for a time-varying electric field.

Figure 3.1 shows that no charge is present at point P. The electric field $\boldsymbol{E}(x)$ produced by charge q passes through P from left to right, so the net flux through P is zero, so P is a source-free point. By contrast, according to Equation (3-2), the divergence of the electric field $\boldsymbol{E}(x)$ is a nonzero negative value, which means that P is a source point that absorbs the electric field lines. Therefore, at point P, the divergence of the electric field $\boldsymbol{E}(x)$ incorrectly describes a source-free point as a source point. Consequently, **at a certain point in space, the nabla-operator divergence of the electric field no longer retains its original physical meaning.**

For one-dimensional fields, the formulas for the divergence and the gradient of the nabla-operator coincide. Correspondingly, by Equation (3-2), the nabla-operator divergence of the electric field $\boldsymbol{E}(x)$ at point P simply represents the rate of change of $\boldsymbol{E}(x)$ along the x-axis. However, in two and three dimensions, the mathematical expressions for divergence and gradient differ. Consequently, , **at a point in space, the nabla-operator divergence does not have a definite mathematical and physical meaning.**

According to the two definitions of divergence in Equation (2-4), the **flux-limit divergence** provides the physical definition of divergence, while the **nabla-operator divergence** serves as a mathematical tool for calculation and derivation. Below, we will compute and verify the flux-limit divergence using the electric field of a point charge.

Refer to the example in Figure 3.1: consider a static point charge q in a vacuum. Select an arbitrary point P at distance $x$ from q, take the point charge q as the origin, and choose the line from q to P as the x-axis. The electric field at any point on the $x$-axis has only an $x$-component, yielding a one-dimensional electrostatic field. Using q as the origin and the $x$-axis as the symmetry axis, construct a conical surface that subtends a solid angle $\Delta\Omega$. This conical surface intercepts two circular surface elements, $\Delta S_1$ and $\Delta S_2$, which are perpendicular to the $x$-axis and whose centers lie at P and $P_+$ respectively; the separation between $\Delta S_1$ and $\Delta S_2$ is $\Delta x$. The conical surface together with $\Delta S_1$ and $\Delta S_2$ encloses a conical frustum, as shown in Figure 3.2.

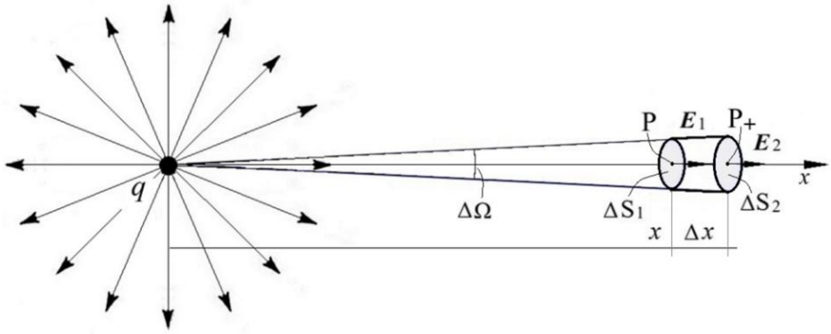


Figure 3.2 Flux-limit divergence of the field of a point charge

Figure 3.2 shows surface elements $\Delta S_1 = \Delta\Omega\, x^2$ and $\Delta S_2 = \Delta\Omega\, (x + \Delta x)^2$. As the solid angle $\Delta\Omega$ approaches to an infinitesimal nonzero value， the electric fields on $\Delta S_1$ becomes uniform and equals the field $\boldsymbol{E}_1$ at the circle's center P. Likewise, the electric field on $\Delta S_2$ equals $\boldsymbol{E}_2$ at the center $P_+$. The net electric flux through the closed surface of the conical frustum is:

$$\boldsymbol{E}_2\Delta S_2 - \boldsymbol{E}_1\Delta S_1 = \left(\frac{q}{4\pi\varepsilon_0(x+\Delta x)^2}\right)\Delta\Omega\,(x+\Delta x)^2 - \left(\frac{q}{4\pi\varepsilon_0 x^2}\right)\Delta\Omega\, x^2$$

$$= \left(\frac{q}{4\pi\varepsilon_0}\right)\Delta\Omega - \left(\frac{q}{4\pi\varepsilon_0}\right)\Delta\Omega = 0$$

The volume of the conical frustum is:

$$\Delta V = \frac{1}{3}\Delta x(\Delta S_1 + \Delta S_2 + \sqrt{\Delta S_1\,\Delta S_2})$$

According to Equation (2-4), the **flux-limit divergence** of the electric field within the infinitesimal region at point P:

$$\lim_{\Delta V\to 0}\frac{\oiint_S \boldsymbol{E}\bullet dS}{\Delta V} = \lim_{\Delta V\to 0}\frac{\boldsymbol{E}_2\Delta S_2 - \boldsymbol{E}_1\Delta S_1}{\Delta V}$$

$$\lim_{\Delta V\to 0}\frac{\oiint_S \boldsymbol{E}\bullet dS}{\Delta V} = \lim_{\Delta V\to 0}\frac{0}{\Delta V} = 0 \qquad (3-3)$$

In Equation (3-3), ΔV denotes an infinitesimal volume element that approaches zero but is not identically zero, so the flux-limit divergence equals to zero. This result agrees with Gauss's flux theorem in integral form. By contrast, in Equation (3-2), the nabla-operator divergence is a nonzero negative value. Consequently, the two definitions in Equation (2-4), **the flux-limit divergence** and **the nabla-operator divergence are not equivalent.**

When the charge density ρ = 0, Gauss's flux theorem in differential form does not hold. **Below, using examples based on Coulomb's law of electrostatics, we demonstrate that Gauss's flux theorem in differential form also does not hold when ρ ≠ 0.**

As shown in Figure 3.3, without loss of generality, consider a slender charged wire as the *x*-axis. The wire is 1000 mm long with radius r = 0.1 mm, and the charge is distributed along the wire with corresponding charge density ρ(*x*). Because the length is much greater than the radius, the electric field ***E***(*x*) at any point on the wire has only an *x*-component, so the field ***E***(*x*) can be treated as a one-dimensional static electric field.

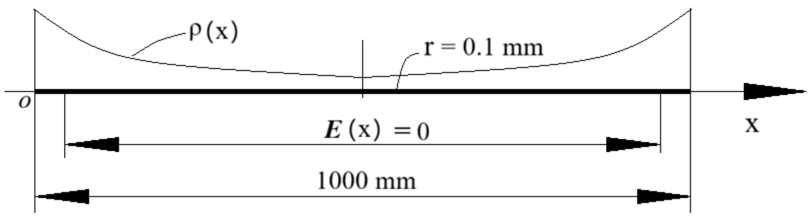


Figure 3.3 Nabla-operator divergence of the field of a charged slender wire

Applying the three-dimensional Gauss's flux theorem in differential form (Equation (1-8B)) to the slender wire, the one-dimensional Gauss's flux theorem in differential form is given as follows:

$$\frac{\mathrm{d}\boldsymbol{E}(x)}{\mathrm{d}x} = \frac{\rho(x)}{\varepsilon_0} \qquad (3-4)$$

Ignoring the electric fields at the wire's ends, the Coulomb force on charges along the wire has only an *x*-component and is balanced, so the electric field ***E***(*x*) along the wire is zero. According to Coulomb's law of electrostatics, the charge distribution on the wire must be nonuniform: the charge density ρ(*x*) is lower in the middle and higher near wire ends [16]. Consequently, ρ(*x*) is nonzero and varies with *x*.

Because the electric field ***E***(*x*) along the slender wire is zero, according to the one-dimensional Gauss's flux theorem in differential form (Equation (3-4)), the nabla-operator divergence of the electric field ***E***(*x*) is:

$$\frac{\mathrm{d}\boldsymbol{E}\,(x)}{\mathrm{d}x} = \frac{\mathrm{d}\,0}{\mathrm{d}x} = 0 \qquad (3-5)$$

Equation (3-5) implies the left-hand side of Equation (3-4) is identically zero, while the charge density ρ(*x*) is nonzero, indicating that the right-hand side of Equation (3-4) is nonzero, so Equation (3-4) does not hold. Consequently, **when the charge density ρ ≠ 0, with nabla-operator divergence, Gauss's flux theorem in differential form does not hold for a static**

**electric field.** By a similar argument, Gauss's flux theorem in differential form does not hold for a time-varying electric field when $\rho \neq 0$.

There are two definitions of divergence. **Below, we will compute and verify the flux-limit divergence based on the charged slender wire.**

Consider an infinitesimal segment of the slender wire shown in Figure 3.3 with length $\Delta x$ and radius r. Denote its volume by $\Delta V$ and the area of its closed surface by $\Delta S$, as illustrated in Figure 3.4.

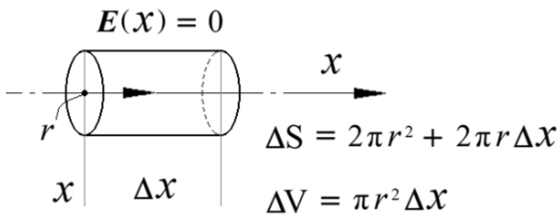


Figure 3.4 Flux-limit divergence of the field of a charged slender wire

According to the three-dimensional Gauss's flux theorem in differential form (Equation (1-8B)) and the two definitions of divergence (Equation (2-4)), applying the flux-limit divergence to the infinitesimal segment of the slender wire, the Gauss flux theorem in differential form is given as follows:

$$\lim_{\Delta V \to 0} \frac{\oiint_S \boldsymbol{E}(x) \cdot dS}{\Delta V} = \frac{\rho(x)}{\varepsilon_0} \qquad (3-6)$$

In Figure 3.3, the electric field $\boldsymbol{E}(x)$ along the slender wire is zero, then the electric flux $\Delta\Phi_E$ through the closed surface surrounding the infinitesimal segment is also zero. As $\Delta V \to 0$ ($\Delta x \to 0$, $r \to 0$), according to Equation (3-6), the flux-limit divergence for this infinitesimal segment is:

$$\lim_{\Delta V \to 0} \frac{\oiint_S \boldsymbol{E}(x) \cdot dS}{\Delta V} = \lim_{\Delta V \to 0} \frac{\Delta\Phi_E}{\Delta V} = \lim_{\Delta V \to 0} \frac{0}{\Delta V}$$

$$\lim_{\Delta V \to 0} \frac{\oiint_S \boldsymbol{E}(x) \cdot dS}{\Delta V} = \lim_{\Delta V \to 0} \frac{0}{\Delta V} = 0 \qquad (3-7)$$

Equation (3-7) implies that the left-hand side of Equation (3-6) is identically zero, while the charge density $\rho(x)$ is nonzero, indicating that the right-hand side of Equation (3-6) is nonzero, so Equation (3-6) does not hold. Consequently, **when the charge density $\rho \neq 0$, with the flux-limit divergence, Gauss's flux theorem in differential form does not hold for a static electric field.** By a similar argument, Gauss's flux theorem in differential form does not hold for a time-varying electric field when $\rho \neq 0$.

Figure 3.3 shows that the electric field $\boldsymbol{E}(x)$ is zero at every point along the slender wire because the electric field at a given point is the superposition of the electric fields produced by all charges distributed on the slender wire. By contrast, Gauss's flux theorem in differential form states that the electric field at a given point is solely dependent on the charge at that point in terms of charge density, disregarding contributions from other charges on the wire. Therefore, **Gauss's flux theorem in differential form contradicts the superposition principle of Coulomb's law.**

Gauss's flux theorem in integral form (Equation (1 - 8A)) implies that no charge resides on the chosen closed surface. By contrast, according to the definitions of divergence in Equation (2-4), the closed surface and its enclosed volume are collapsed to a single point in space, which places the charge on the closed surface. Consequently, **the integral and differential forms of Gauss's flux theorem appear mutually contradictory and therefore not equivalent.**

Coulomb's law requires a nonzero separation r between a point charge and the electric field test point. By contrast, in the differential form of Gauss's flux theorem (Equation (1-8B)), the point charge, in terms of charge density, and the electric field test point are the same spatial point, which corresponds to r = 0. Consequently, **Gauss's flux theorem in differential form conflicts with the constraints of Coulomb's law.**

Gauss's flux is the surface integral of the electric field over a closed three-dimensional surface, representing a macroscopic property of the electric field within a specified region. When Gauss's flux, characterized by three-dimensional space, is compressed to a single point, the divergence no longer preserves the original physical meaning of the flux. The divergence of one-dimensional electric fields in Equations (3-2) and (3-5) does not signify a source or source-free characteristic of the electric fields; instead, it indicates the rate of change of the electric field's magnitude along the *x*-axis. However, in a three-dimensional electric field, divergence does not indicate the rate of change of the electric field's magnitude in space and lacks a clear mathematical and physical meaning. Therefore, **the divergence of an electric field at a point has no definite mathematical and physical significance.**

By a similar argument, the magnetic flux theorem in differential form, which relies on the divergence of a magnetic field at a point, does not hold in both static and time-varying magnetic fields, and has no definite mathematical and physical significance.

## 4. Summary and Outlook

Before 1850, classical electromagnetism had been basically established. Based on divergence, curl, and nabla-operator, Maxwell's equations are an integration of partial differential equations and classical electromagnetism, **they are not part of classical electromagnetism.**

In classical electromagnetism, Gauss's flux theorem in integral form is originally derived from Coulomb's law of electrostatics. In the static electric fields, an electric field entering a closed surface is equal to that leaving it. By contrast, in the time-varying electric fields, an electric field entering a closed surface is not equal to that leaving it. **Gauss's flux theorem in integral form applies to static electric fields but not to time-varying electric fields.**

Gauss's flux theorem in integral form implies that no charge resides on the chosen closed surface. By contrast, according to the definitions of divergence, the closed surface and its enclosed volume are collapsed to a single point in space, which places the charge on the closed surface. T**he differential and integral forms of Gauss's flux theorem appear mutually contradictory and therefore not equivalent.** Coulomb's law requires a nonzero separation r between a point charge and the electric field test point. However, in the differential form of Gauss's flux theorem, the point charge, represented by a charge density, and the electric field test point are the same spatial point, which corresponds to r = 0. Therefore, **Gauss's flux theorem in differential form conflicts with the constraints of Coulomb's law.** The superposition principle states that the electric field at a given point is the vector sum of the fields produced by all charges in space. By contrast, Gauss's flux theorem in differential form states that the electric field at a given point is solely dependent on the charge at that point in terms of charge density, disregarding contributions from other charges in space. Consequently, **Gauss's flux theorem in differential form contradicts the superposition principle of Coulomb's law.**

In conclusion, the differential and integral forms of Gauss's flux theorem are not equivalent, and the differential form conflicts with the constraints of Coulomb's law. Gauss's flux theorem in differential form, which depends on the divergence of an electric field at a point, does not hold in both static and time-varying electric fields, and has no definite mathematical and physical significance.

Gauss's flux is the surface integral of electric fields over a three-dimensional closed surface and thus represents a macroscopic property of the electric field within a specified region. When Gauss's flux, characterized by three-dimensional space, is compressed to a single point, the divergence no longer preserves the original physical meaning of the flux. The divergence of one-dimensional electric fields does not signify a source or source-free characteristic of the electric fields; instead, it indicates the rate of change of the electric field's magnitude along the $x$-axis. However, in a three-dimensional electric field, divergence does not indicate the rate of change of the electric field's magnitude in space. **Therefore, the divergence of a vector at a point in space has no definite mathematical and physical significance.**

In the integral form of Ampère's circuital law, Ampere's circulation is the line integral of the magnetic field along a closed curve and thus represents a macroscopic physical property of the magnetic field within a specified region. When Ampere's circulation, characterized by three-dimensional space, is compressed to a single point, the **curl no longer preserves the original meaning of the circulation， and has no definite mathematical and physical significance** [17].

The time-varying electric and magnetic fields in electromagnetics, and the time-varying velocity fields in fluid mechanics are all vector traveling waves [18]. As a vector traveling wave propagates, its waveform translates along the propagation direction over time. In general, a traveling wave $\boldsymbol{F}(x,\ t)$ varies sinusoidally in time, it also varies sinusoidally in space. **For a one-dimensional sinusoidally varying vector traveling wave, the mathematical representation can be expressed as:**

$$\boldsymbol{F}(x,t) = \boldsymbol{F}_0 + \boldsymbol{F}_{\mathrm{C}}\,\mathrm{Sin}(\omega t - \frac{\omega}{c_w} x)$$

Here, $\boldsymbol{F}_0$ denotes the static component of the vector; $\boldsymbol{F}_{\mathrm{C}}$, the sine-wave peak amplitude; $\omega/c_w$, the phase-shift coefficient; $\Delta\varphi = x\ \omega/c_w$,the phase difference; $\omega$, the angular frequency; and $c_w$, the traveling-wave velocity (phase velocity).

The vector traveling wave $\boldsymbol{F}(x,\ t)$ is a function of time and space. We must therefore determine its time derivative and its spatial rate of change. At any point in space the wavevector has both magnitude and direction. The divergence and curl have no definite mathematical and physical significance; new mathematical tools are required to describe (i) the spatial rate of change of the vector magnitude along the propagation direction and (ii) the angular rate of change of the vector direction about its rotation center. **Developing such tools represents a noteworthy opportunity in mathematical physics.**

Divergence and curl are the integration of mathematics and physics. At a single point in space, the divergence and curl of a vector have no definite mathematical and physical significance. **Theoretical physics and mathematics will undergo a revolutionary scientific transformation.**

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