Wave Mechanics Equation with Explicit Force

Runsheng Tu
Independent scholar, China
(ORCID: 0000-0001-5349-8007)

Abstract

Classical mechanics and quantum mechanics are considered to be binary oppositions. This kind of understanding has never brought us any real benefits, only trouble. The fundamental equations of quantum mechanics cannot logically delineate the boundary between quantum and classical mechanics. On the contrary, the mass m and potential energy V in the Schr ö dinger equation can easily cross the boundary between microscopic and macroscopic levels. The electromagnetic potential energy in the Schrödinger equation can be replaced with the potential energy of the interaction force. Potential energy can be further transformed into force based on *F*=*V*/*r*. This operation can establish Newton's second law wave equation (e.g., $\hat{F} = -i\hbar\frac{\partial^2}{\partial t \partial x}$) and force containing wave mechanics equation (*e.g.*, $-\alpha c\hbar\frac{\partial^2}{\partial x^2}\psi + \frac{Ze^2}{2r^2}\psi = F\psi$). This result more directly indicates that quantum and classical are compatible. Eliminating the historical absence of the concept of "force" in quantum mechanics, bridging the gap between macroscopic classical and microscopic quantum frameworks, and changing people's perceptions. Can explain why nanoscale particles have quantum properties. Simplify the calculation of quantum forces. This lays the foundation for the generalized wave mechanics that unifies mechanics across scales.

Keywords: Quantum and classical, Compatibility, The wave mechanics equation of Newton's second law, Quantum mechanics operator of force, Schrödinger-Tu equation with explicit force.

1. Intraduction

For a long time, classical mechanics and quantum mechanics have been regarded as irreconcilable binary oppositions in physical theoretical systems. The former describes macroscopic systems with deterministic orbits and observable forces as the core, while the latter relies on wave function probability interpretation and operator form to handle microscopic phenomena. The significant differences in conceptual frameworks and mathematical tools between the two have led to an artificial disconnect between "macroscopic classicism" and "microscopic quantum". Fortunately，this study suggests that this opposition is not essential—classical mechanical laws can be reformulated through wave mechanics equations, achieving deep compatibility with quantum mechanics. The core technological path is to use the momentum operator conversion method of quantum mechanics, combined with the intrinsic relationship between force and energy in planetary models, to derive wave equations containing "force", thereby truly returning quantum mechanics to the origin of "mechanics" and ending the historical lack of the concept of "force" in quantum mechanics.

The author has systematically demonstrated the compatibility between quantum and classical mechanics in the previous series of studies, and proposed a theoretical framework for the synergistic application of quantum mechanics methods and classical mechanics methods. [1-14] As a deepening and expansion of the previous work, this article further constructs the wave mechanics equations of classical mechanics core laws (such as the law of conservation of momentum and Newton's second law), and for the first time derives quantum operator representations of physical quantities such as force and acceleration.

At the beginning of the birth of quantum mechanics, physicists hoped for its compatibility and unity with classical mechanics. However, due to the ambiguity of the physical meaning of wave functions and the difficulty in explaining phenomena such as double slit diffraction experiments and hydrogen atom stability, the mainstream view has gradually shifted towards denying the "the independent existence of primitive eigenstates the existence of primitive eigenstate" of microscopic particles that are unrelated to measurement from a philosophical perspective. The uncertainty, randomness, and "wave function collapse" of microscopic systems are regarded as inherent properties of quantum mechanics, and it is asserted that the Schrödinger equation cannot describe macroscopic objects. But in fact, the Schrödinger equation itself does not have an inherent limitation that logically excludes macroscopic systems—when the mass m is extended to the macroscopic observable scale and the potential energy function is replaced with macroscopic gravitational potential energy, its mathematical form still holds. [1] This article will reinforce this conclusion through more intuitive deduction.

Although the academic community has never stopped questioning the Copenhagen interpretation,[15-25] there is less controversy over the validity of the mathematical formal system of quantum mechanics. This study also does not deviate from the axiomatic system of quantum mechanics, but strictly follows the construction rules of Hermitian operators ($\langle\phi|\hat{A}|\psi\rangle = \langle\psi|\hat{A}\phi|\rangle$) and derives wave mechanics equations containing "force" without introducing new assumptions. Previous studies (Especially combining or alternating the use of quantum mechanics and classical mechanics) [26-30] have provided multidimensional evidence of quantum classical compatibility, weakening the opposition between the two and confirming the advantages of compatibility frameworks in theory and applications. Previously, the combination of quantum and classical methods mainly relied on the parallel or alternating use of traditional methods (such as replacing only the potential energy function without changing the structure of the quantum equation); The breakthrough of this article lies in integrating quantum and classical properties into the same equation, deriving the generalized wave mechanics equation, laying the foundation for the establishment of "generalized wave mechanics".

2. Deduction of "Operator of Force" and "Wave Equation of Classical Mechanics Laws"

The general form of the one-dimensional stationary Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+V\psi=E\psi. \qquad (1)$$

Since the calculation result of $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi$ is the kinetic energy $T\psi$, therefore, Eq. (1) eliminates psi to obtain $T+V=E$. $T+V=E$ is a universal formula for energy conservation. Both microsystems and macrosystems are established. It can also satisfy the energy allocation ratio of T and V in bound state systems (*i.e.*, satisfy the Viry theorem under certain conditions). It can be seen that Eq. (1) is also a wave mechanics expression that describes the classical laws of physics for macroscopic phenomena. When $V=-\frac{Ze^2}{r}$, Eq. (1) can describe the electromagnetic interaction planetary model system. When the potential energy V is $V=-\frac{GMm}{r}$, Eq. (1) can be used to describe the Newtonian gravitational planetary model system. It can be seen that the Schrödinger equation itself cannot deny its ability to describe macroscopic gravitational interaction systems. In the case where V is the gravitational potential energy, the Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi-\frac{GMm}{r}\psi=E\psi. \text{ [1-9]} \qquad (2)$$

The wave function should still use the familiar form:

$$\psi = \psi_0 e^{-\frac{i}{\hbar}(Et-px)}= \psi_0 e^{-i2\pi(\nu t-x/\lambda)}. \qquad (3)$$

Whether it is a macroscopic gravitational interaction planetary model system or a microscopic electromagnetic interaction planetary model system, the relationship between potential energy and force is $F=V/r$. In the planetary model, the relationship between E, V, υ, r, and F is $E=\frac{1}{2}V=-\frac{1}{2}m\upsilon^2=\frac{1}{2}rF$.

The energy conservation of a steady-state system satisfies

$T+V=E. \qquad (4)$

Aamong which, $E=-\langle T\rangle$ (Its numerical relationship is $E=-T$). By substituting $E=-T$ into Eq. (4), result in $2T=V$. By dividing both sides of the equation by the orbital radius r in the planetary model, and considering F=V/r, we can obtain the relationship between kinetic energy and force: $2T/r=F$. Substituting $T=mv^2/2$ into $2T/r=F$, and considering that the acceleration of an object in uniform circular motion is $a=v^2/r$, we can obtain Newton's Second Law: $\frac{mv^2}{r}=ma=F$.

The formula for kinetic energy in classical mechanics is $T=\frac{p^2}{2m}$. For an object in uniform circular motion, $2T/r=F$ can be written as $F=\frac{mv^2}{r}$. Multiply both sides of $F=mv^2/r$ by $\frac{1}{2}r$, we can obtain $\frac{1}{2}Fr=\frac{1}{2}mv^2=T$. Substituting $\frac{1}{2}mv^2=T$ into Eq. (4), result in $(1/2)mv^2+V=E$, or $(1/2)mv^2+Fr=E$. We write the classical mechanical formula $(1/2)mv^2+Fr=E$ *as*

$$\frac{p^2}{2m}+V=E. \qquad (5)$$

If all terms of Eq. (5) are multiplied by ψ, we can obtain $\frac{p^2}{2m}\psi+V\psi=E\psi$. Considering Eq. (3), we have $\frac{p^2}{2m}\psi=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=T=\frac{1}{2}mv^2\psi$. In the planetary model of hydrogen atoms, the orbital angular momentum of electrons is $L=pr=\frac{1}{2}\hbar$. The orbital velocity of $n=1$ electron in the ground state hydrogen atom is $v=\alpha c$.

If equation (1) is used, that is, V takes $-\frac{Ze^2}{r}$, and both sides of the equation are divided by r, then $F=V/r$ is the electromagnetic interaction force. We have

$$\frac{i\hbar^2}{2m}\frac{\partial^3}{\partial x^3}\psi-\frac{Ze^2}{r^2}\psi=\frac{E}{r}\psi. \qquad (6)$$

For the hydrogen atom in planetary models, $V=2E$, Therefore we have $\frac{i\hbar^2}{2m}\frac{\partial^3}{\partial x^3}\psi+\frac{V}{2r}\psi=F\psi$, or

$$\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}\psi-\frac{Ze^2}{r^2}\psi=2F\psi. \qquad (7)$$

In the planetary model of hydrogen atoms, the orbital angular momentum of electrons is $L=pr=\frac{1}{2}\hbar$. The orbital velocity of $n=1$ electron in the ground state hydrogen atom is $v=\alpha c$. If we want it to be in the form of a second partial derivative, then the equation corresponding to equation (7) is

$$-\alpha c\hbar\frac{\partial^2}{\partial x^2}\psi+\frac{Ze^2}{2r^2}\psi=F\psi. \qquad (8)$$

Hear, α is the fine structure constant. For gravitational interactions, it is well known that $-\frac{GMm}{r^2}=F$, $L=mvr=m\sqrt{GMr}$, and according to equation (2), we can obtain

$$\frac{\hbar^4}{2GMm^3}\frac{\partial^4}{\partial x^4}\psi-\frac{GMm}{r^2}\psi=\frac{E}{r}\psi. \qquad (9)$$

Here, $E=\frac{1}{2}V$, $\frac{E}{r}\psi=\frac{V}{2r}\psi$. Substituting $F=-\frac{GMm}{r^2}$ into equation (9), we can obtain

$$-\frac{\hbar^4}{2GMm^3}\frac{\partial^4}{\partial x^4}\psi+\frac{V}{2r}\psi=F\psi. \qquad (10)$$

or

$$-\frac{\hbar^4}{GMm^3}\frac{\partial^4}{\partial x^4}\psi+\frac{V}{r}\psi=2F\psi. \qquad (11)$$

We calculate equation (7) item by item. As long as you note that $2pr$=$\hbar$, the calculation results of the two terms on the left side of the equal sign are: $\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}\psi = \frac{p^2}{mr}\psi = \frac{2T}{r}\psi = -\frac{V}{r}\psi = -\frac{V}{r}\psi$; $\frac{Ze^2}{r^2}\psi$=$-\frac{V}{r}\psi$. The sum of these two calculation results is $-\frac{2V}{r}\psi = 2F\psi$. It has been proven that equation (7) holds true. Let's calculate equation (10) item by item. Just pay attention mr=$mGMr$ (*i. e.*, v^2=GM/r), $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = T\psi = \frac{1}{2}mv^2\psi$ and $\frac{\partial^4}{\partial x^4}\psi = \frac{p^4}{\hbar^4}\psi$, we have $-\frac{\hbar^4}{2GMm^3}\frac{\partial^4}{\partial x^4}\psi = -\frac{mv^2}{2r}\psi = \frac{V}{2r} = \frac{1}{2}F\psi$. The second term of equation (10) is also $\frac{1}{2}F\psi$. The sum of two $\frac{1}{2}F\psi$ is exactly the $F\psi$ on the right. So, equation (10) holds true.

If all terms of F=ma are divided by the interaction distance r, and considering F=dV/dr=−dp/dt, especially its non differential formula, we can obtain $F = -\frac{GMm}{r}$ or $F = \frac{mv^2}{r}$. Note: we tend to use negative values to represent potential energy and force (In the equation F=ma, the direction of F is consistent with the direction of acceleration a). For the uniform circular motion of electrons, pr=$\frac{1}{2}\hbar$. Considering p=mv, F=V/r and "the gravitational force in a circular motion equilibrium system is equal to the centrifugal force", we can obtain

$$F=-2p^3/m\hbar. \qquad (12)$$

As is well known, $\hat{p} = -i\hbar\frac{\partial}{\partial x}$, so, the operator of p^3 is

$$\hat{p}^3 = i\hbar^3\frac{\partial^3}{\partial x^3}. \qquad (13)$$

By replacing p^3 in equation (12) with Eq. (13), we can obtain

$$\hat{F} = -2\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}. \qquad (14)$$

Multiply both sides of E. (14) by ψ to obtain equation (15). It is the representation method of wave power of force.

$$\hat{F}\psi = -2\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}\psi. \qquad (15)$$

In the equilibrium system of bound state mechanics, the force exerted by the atomic nucleus on the electron conforms to this equation. For the Bohr planetary model, pr=$\hbar$, the negative sign on the right side of equations (14) and (15) should be removed. This article believes in the planetary model of hydrogen atoms and does not believe in the electron cloud hydrogen atoms with zero orbital angular momentum in quantum mechanics. As is well known, the operator of kinetic energy T is

$$\hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}. \qquad (16)$$

In classical mechanics, the expression for the law of conservation of momentum is m_1v_1=m_2v_2 or p_1=p_2. By replacing p with the corresponding operator in p_1=p_2, the result is $-i\hbar\frac{\partial}{\partial x_1} = -i\hbar\frac{\partial}{\partial x_2}$. Adding corresponding wave functions $\psi_1\psi_2$ on both sides of this result, we can obtain $-i\hbar\psi_2\frac{\partial}{\partial x_1}\psi_1$=$-i\hbar\psi_1\frac{\partial}{\partial x_2}\psi_2$, or

$$-\frac{i\hbar}{\psi_1}\frac{\partial}{\partial x_1}\psi_1=-\frac{i\hbar}{\psi_2}\frac{\partial}{\partial x_2}\psi_2. \qquad (17)$$

Here, $\psi_1 = A_1e^{-\frac{i}{\hbar}(E_1t_1-p_1x_1)}$, $\psi_2 = A_2e^{-\frac{i}{\hbar}(E_2t_2-p_2x_2)}$, $E_1 = h\nu_1$, $E_2 = h\nu_2$. Eq. (17) is the law of conservation of momentum expressed using wave mechanics methods.

Calculate the first term in Eq. (1) and move some terms to become $\frac{p^2}{2m}\psi - E\psi = -V\psi$. Considering $E=-T$, $V=-2T$, we have

$$-\frac{p^2}{m}\psi = V\psi. \qquad (18)$$

By replacing p^2 in Eq. (18) with an operator form, and divide both sides of equation (18) by r, the potential energy operator $\hat{V} = \frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}$ can be obtained (note: $a=\upsilon^2/r$, potential energy V is a negative value).

$$\frac{p^2}{mr}\psi = F\psi = \frac{p}{\upsilon}\frac{\upsilon^2}{r}\psi. \qquad (19)$$

Concidering $m\upsilon=p$, $a=\upsilon^2/r$, Eq. (19) becomes $F\psi=[\frac{p}{\upsilon}a]\psi$. By replacing the momentum p with the operator $\hat{p}=-i\hbar\frac{\partial}{\partial x}$ and using the relationship $\upsilon=c\alpha$ between velocity and fine constant in the ground state Bohr hydrogen atom, we can obtain

$$\hat{F}\psi = -a[\frac{i\hbar}{c\alpha}\frac{\partial}{\partial x}]\psi. \qquad (20)$$

Equation (20) is one of the wave mechanics expressions for Newton's second law applicable to the ground state Bohr hydrogen atom. If particles move uniformly in a straight line, their form will be different.

For de Broglie waves in planetary models, $\lambda=h/m\upsilon=h/p$, $\upsilon=\lambda\nu$ and $a=\upsilon^2/r$. Using these telationship, the first equation in Eq. (19) can be transformed into a $a\left[\frac{p^2}{h\nu}\right]\psi = F\psi$. Reference [1] has the following equation: $T=\frac{1}{2}h\nu$. Use this equation, $a\left[\frac{p^2}{h\nu}\right]\psi = F\psi$ can be transformed into $a\left[\frac{p^2}{2T}\right]\psi = F\psi$. By replacing p^2 with the corresponding operator $\hat{p}^2 = -\hbar^2\frac{\partial^2}{\partial x^2}$, we can obtain

$$\hat{F}\psi = -a\left[\frac{\hbar^2}{2T}\frac{\partial^2}{\partial x^2}\right]\psi. \qquad (21)$$

Equation (21) has a wide range of applicability (applicable to both micro and macro objects, as well as objects in linear motion). This is different from equation (20). According to it, a new operator of force can be obtained: $\hat{F} = -\frac{a\hbar^2}{2T}\frac{\partial^2}{\partial x^2}$ (where a is acceleration).

Comparing $F=ma$ with $\hat{F} = -\frac{a\hbar^2}{2T}\frac{\partial^2}{\partial x^2}$, we can obtain the operator of the mass m as a function.

$$\hat{m} = -\frac{\hbar^2}{2T}\frac{\partial^2}{\partial x^2}. \qquad (22)$$

According to Eqs. (20) and (21), Eq. (7) can be transformed to

$$-a\left[\frac{\hbar^2}{4T}\frac{\partial^2}{\partial x^2}\right]\psi - \frac{E}{r}\psi = F\psi. \qquad (23)$$

According to $\hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$, $\hat{V}=\frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}$, $E=-T=-\frac{p^2}{2m}$, $\hat{E}=-\hat{T}=\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}=-i\frac{\hbar}{2}\frac{\partial}{\partial t}$ (the last equation can be found in Refs. [1-9]), the wave mechanics expression of Viry's theorem can be obtained: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + \frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}\psi = -i\frac{\hbar}{2}\frac{\partial}{\partial t}\psi$ or

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi = -i\frac{\hbar}{2}\frac{\partial}{\partial t}\psi. \qquad (24)$$

The concise form of the classical expression for Viry's theorem is $V=-2T$. Considering $\hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$, we have $V\psi =$

$\frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}\psi$.

According to the one-dimensional momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$, We can directly write the velocity operator, velocity squared operator, and acceleration operator.

$$\hat{\upsilon}=\hat{\dot{r}}=-\frac{i\hbar}{m}\frac{\partial}{\partial x}, \qquad (25)$$

$$\hat{\upsilon}^2=-\frac{\hbar^2}{m^2}\frac{\partial^2}{\partial x^2}. \qquad (26)$$

$$\hat{a} = \hat{\dot{\upsilon}} = -\frac{\partial}{\partial t}\left[\frac{i\hbar}{m}\frac{\partial}{\partial x}\right]. \qquad (27)$$

The displacement equation of the object in motion is $s=\upsilon t$. According to equation (25), we know that $\hat{s} = -i\hbar\frac{t}{m}\frac{\partial}{\partial x}$ and $s = -i\hbar\frac{t}{m}\frac{\partial}{\partial x}\psi$. Quantum mechanics also uses the momentum relationship $p=m\upsilon$. Based on the derivative of the wave function of kinetic energy with respect to t, and considering $m=2T/\upsilon^2$, the quality operator is $\hat{m}=-i\hbar\frac{\partial}{\partial t} \div \left[-\frac{\hbar^2}{m^2}\frac{\partial^2}{\partial x^2}\right]$, and the result is

$$\hat{m} = i\frac{m^2}{\hbar}\left[\frac{\partial}{\partial t}/\frac{\partial^2}{\partial x^2}\right]. \qquad (28)$$

As long as all classical physical quantities have wave mechanics operators, the wave mechanics expressions of classical physical laws can be directly combined by the corresponding operators. For example, the operator formula with constant mass for Newton's second law $F=ma$ is $\hat{F}=m\hat{a}$. The result of the operation is Eq. (28).

$$\hat{F}=-i\hbar\frac{\partial^2}{\partial t\partial x}. \qquad (29)$$

Replacing the momentum in $F=\partial p/\partial t$, with an operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$, Eq. (29) can be directly obtained. By applying equation (29) to the wave function, we can obtain

$$\hat{F}\psi=-i\hbar\frac{\partial^2}{\partial t\partial x}\psi. \qquad (30)$$

In Eq. (30), the calculation method for $\frac{\partial^2}{\partial t\partial x}\psi$ is to first calculate the partial derivative of ψ with respect to x, then calculate the partial derivative of ψ with respect to t, and finally calculate the product of the two partial derivatives. Equation (30) is the universal Newton's second law representated by using wave equations.

The three-dimensional form of Eq. (8)

$$-c\alpha\hbar\nabla^2\psi + \frac{Ze^2}{2r}\psi = F\psi. \qquad (31)$$

Equation (30) is applicable to hydrogen atoms. After writing equation (30) as $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{E}{r}\psi = F\psi$, it is already very similar to the Schrödinger equation. The first term in equation (30) is similar to the first term in the Dirac equation. 我们 The time-dependent Schrödinger equation can also be transformed into a wave mechanics equation with explicit force. The transformation equation is $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{i\hbar}{2}\frac{\partial}{\partial t}\psi$. Using this transformation method, the Schrödinger equation must be corrected. The corrected time-dependent Schrödinger equation is

$$-i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi. \qquad (32)$$

Dividing both sides of equation (31) by *r*, we can obtain

$$-\frac{\mathrm{i}\hbar}{r}\frac{\partial}{\partial t}\psi + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = F\psi. \qquad (33)$$

For the planetary model hydrogen atom, according to $pr = \frac{1}{2}\hbar$, $1/r = 2p/\hbar$. $v = \alpha c$, we can obtain

$$-i2m\alpha c\frac{\partial}{\partial t}\psi + \alpha c\hbar\frac{\partial^2}{\partial x^2}\psi = F\psi. \qquad (34)$$

3. Discussion

The present study demonstrates that classical mechanics and quantum mechanics are not mutually exclusive but can be unified through wave equation representations, addressing a longstanding divide in physical theory. By deriving wave mechanics expressions for classical laws—including Newton's second law, the law of conservation of momentum, and the virial theorem—we bridge microscopic and macroscopic systems, particularly within planetary models.

3.1. Key Findings and Implications

Force Operators in Quantum Mechanics: A central contribution is the derivation of force operators [(Eqs. (14) and (25)] and wave equations incorporating force [Eqs. (7)-(11), (23), (24) and (30)], challenging the historical absence of "force" in quantum mechanics. These operators enable direct application of classical force concepts to quantum systems, such as the hydrogen atom, where Eqs. (15) and (30) generalize Newton's second law to wave mechanics.

Unified Planetary Model Descriptions: By modifying the Schrödinger equation to include gravitational or electromagnetic potential energy (Eqs. (2)–(3)), we show that both microscopic (*e.g.*, hydrogen atoms) and macroscopic (*e.g.*, planetary motion) systems can be described using identical mathematical frameworks. This contradicts the conventional view that quantum probability and uncertainty are exclusive to the microscale, suggesting instead that such interpretations may stem from limited application of wave mechanics to macroscopic contexts.

Operator Formalism for Classical Laws: The conversion of classical observables (momentum, velocity, acceleration) into quantum operators [Eqs. (25)–(27)] allows classical laws to be expressed as wave equations. For example, the law of conservation of momentum [Eq. (17)] and Newton's second law [Eq. (30)] retain their physical meaning while adopting wave function-based formulations. This formalism extends to non-constant mass systems [Eq. (28)], broadening applicability to bound-state equilibrium systems.

3.2. Limitations and Future Work

Although this work unifies classical mechanics and quantum mechanics in planetary models, the following limitations are worth exploring.

Generalization Beyond Planetary Models: The derived equations rely on the virial theorem and circular motion assumptions, restricting applicability to systems with radial symmetry. Extending these results to non-planetary, non-bound systems (*e.g.*, scattering or chaotic motion) remains a critical next step. The Schrödinger equation also began with the Bohr planetary model. In the context of planetary models, it is also only applicable to Bohr hydrogen atoms. However, after denying the planetary model, people intuitively believe that it is applicable to all microscopic systems. The physical mechanism behind this is still unclear. This article encountered the same situation (the establishment of equations started from classical mechanics and planetary models, and the application of equations faces a wider range). we don't know if they follow the same physical mechanism. Does the quality operator really obtain the quality as a non constant? It is not explicitly reflected in the paper. This also deserves further research.

Experimental Validation: Empirical testing of force operators in quantum systems, such as measuring force-induced wave function perturbations in atomic orbitals, is necessary to confirm theoretical predictions. Operator Algebra Consistency: The introduction of "heavy partial derivatives" and operator division [*e.g.*, Eqs. (24)-(26)] requires rigorous mathematical validation to ensure consistency with existing quantum operator theory. Collaboration with mathematicians to formalize these operations is essential.

Quantum decoherence, superposition collapse, and wave function flattening are all patches that were later applied, and are not logical results of the mathematical system of quantum mechanics. Quantum and classical are coordinated and compatible, resulting in no clear boundary between quantum and classical. It will seriously affect the credibility of quantum decoherence and wave function collapse. The research findings of this article will lead to a renewed discussion of the concept of quantum decoherence and theoretical patches such as quantum state collapse (wave function collapse or flattening).

4. Conclusion

This research provides a framework for reconciling classical and quantum mechanics through wave equation unification. By reintroducing force into quantum formalism and demonstrating the universality of wave mechanics across scales, we pave the way for a more integrated physical theory. Future work will focus on generalizing these results to diverse systems and validating predictions experimentally, potentially reshaping our understanding of mechanics at both micro and macro levels.

References

[1] Tu, Runsheng. Reasons and consequences for the combination of quantum and classical theory. *Brazilian Journal of Science*, 6(2), 1-28, (2026). https://periodicos.cerradopub.com.br/bjs/article/view/929

[2] Runsheng Tu. Research Progress on the Schrödinger Equation that Can Describe the Earth's Revolution and its Applications, London Journal of Research in Science: Natural & Formal. 25(1) , 17-28 (2025). https://journalspress.com/LJRS_Volume25/Research-Progress-on-the-Schrodinger-Equation-that-Can-escribe-the-Earths-Revolution-and-its-Applications.pdf

[3] Tu, R. Research Progress on the Schrödinger Equation of Gravitational Potential Energy. *Adv Theo Comp Phy*, 8(1), 01-07 (2025). https://dx.doi.org/10.33140/ATCP.08.01.01

[4] Tu. R. Schrödinger-Tu Equation: A Bridge Between Classical Mechanics and Quantum Mechanics. *Int J Quantum Technol*, 1(1), 01-09 (2025). https://www.primeopenaccess.com/scholarly-articles/schrodingertu-equation-a-bridge-between-classical-mechanics-and-quantum-mechanics.pdf

[5] Tu, R. Quantum Mechanics and Classical Mechanics Can Be Used Together. Adv Theo Comp Phy, 8(4), 01-07 (2025). https://doi.org/10.33140/ATCP.08.04.02

[6] Runsheng Tu. Quantum Mechanics and Classical Mechanics Can Be Used Together. Preprint at *viXra*. (2025). https://vixra.org/pdf/2505.0059v3.pdf

[7] Runsheng Tu. Add wings of force to quantum mechanics. Zhihu. (2025). https://zhuanlan.zhihu.com/p/1958532715482711659

[8] Tu, R. A New Theoretical System Combinating Classical Mechanics and Quantum Mechanics. Adv Theo Comp Phy, 8(2), 01-06 (2025). AGR: https://doi.org/10.33140/ATCP.08.02.01

[9] Runsheng Tu. The mutual conversion between Schrödinger equation and *F=ma* visibility. Applied Science Research Periodicals. 4(1): 158-172 (2026) https://doi.org/10.63002/asrp.401.1279

[10] Runsheng Tu. Preliminary application of non-point electron model in structural chemistry. *Abstract Collection of*

Papers from Physical Chemistry Academic Conferences in Henan, Hunan, and Hubei Provinces. 237-238 (1989).
[11] Tu, R. A Wave-Based Model of Electron Spin: Bridging Classical and Quantum Perspectives on Magnetic Moment. Adv Theo Comp Phy, 7(4), 01-10 (2024). DOI: https://dx.doi.org/10.33140/ATCP.07.04.11
[12] Tu, R. A Review of Research Achievements and Their Applications on the Essence of Electron Spin. Adv Theo Comp Phy, 7(4), 01-19(2024). DOI: https://dx.doi.org/10.33140/ATCP.07.04.10
[13] Tu, Runsheng. Solving the problem of source of electron spin magneticmoment. *Physical Science International Journal*. 28(6): 105-110 (2024). DOI: https://doi.org/10.9734/psij/2024/v28i6862
[14] Tu, R. Establishing the Schrödinger Equation for Macroscopic Objects and Changing Human Scientific Concepts. *Adv Theo Comp Phy*, 7(4), 01-03 (2024). https://dx.doi.org/10.33140/ATCP.07.04.18

[15] Sean Carroll. Why even physicists still don't understand quantum theory 100 years on. *Nature*. 638, 31-34 (2025). doi: https://doi.org/10.1038/d41586-025-00296-9
[16] Lee Billings. Quantum Physics Is on the Wrong Track, Says Breakthrough Prize Winner Gerard 't Hooft. Breakthrough Prize Winner Gerard't Hooft Says Quantum Mechanics Is 'Nonsense'. *Scientific American*. 34(2), 104 (2025).
doi: 10.1038/scientificamerican062025-3ndxLHOdGJV2u6twkhWFzi
https://www.scientificamerican.com/article/breakthrough-prize-winner-gerard-t-hooft-says-quantum-mechanics-is-nonsense/
[17] Gibney, E. Physicists disagree wildly on what quantum mechanics says about reality, Nature survey shows. *Nature*. 643, 1175-1179 (2025). https://doi.org/10.1038/d41586-025-02342-y

[18]. Sivasundaram, S. & Nielsen, K. H. Preprint at arXiv, https://doi.org/10.48550/arXiv.1612.00676 (2016).

[19]. Hallas, A. M. Nature Phys. 21, 491–493 (2025).
Article Google Scholar
[20]. Sharoglazova, V., Puplauskis, M., Mattschas, C., Toebes, C. & Klaers, J. Nature 643, 67–72 (2025).

Article PubMed Google Scholar
[21] Einstein. Einstein's Letter to Max Born, 1926.
[22] Schrödinger. The State of Quantum Mechanics, (1935)
[23] David Bohm. The Challenge of Hidden Variable Theory. Bohm's Quantum Theory, 1951
[24] John Bell. On the Einstein-Podolski-Rosen Paradox, 1964
[25] Roger Penrose. The Emperor's New Brain, 1989.

[26] Runsheng Tu. The principle and application of experimental method for measuring the interaction energy between electrons in atom. *International Journal of Scientific Reports*. 2(8), 187–200 (2016). DOI: 10.18203/issn.2454-2156.intjscirep20162808
[27] Tu, R. Some Successfull Applications for Local-Realism Quantum Mechanics: Nature of Covalent-Bond Revealed and Quantitative Analysis of Mechanical Equilibrium for Several Molecules. Journal of Modern Physics. 5(6), 309-318 (2814). DOI: 10.4236/jmp.2014.56041
[29] Runsheng Tu. Progress and Review of Applied Research on New Theory of Electronic Composition and Structure. Infinic energy, 167: 57-71(2024).
[30] Tu, R. Preliminary application of non-point electron model in structural chemistry. Abstract Collection of Papers from Physical Chemistry Academic Conferences in Henan-Hunan-Hubei Provinces, 237-238 (1989).

[31] Tu, R. (2026). The Expression of Classical Mechanics Laws Using Wave Mechanics Equations Reflects the Compatibility Between Quantum and Classical. Adv Theo Comp Phy, 9(3), 01-04.

[32] Tu, Runsheng (2026). Wave Mechanics Equation with Explicit Force. figshare. Dataset. https://doi.org/10.6084/m9.figshare.33164309