Multi-Metric Wormholes: A No-Horizon Theorem and $1/N$ Suppression of Exotic Matter

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June 11, 2026

Abstract

We present a comprehensive study of traversable wormholes in multi-metric gravity, a ghost-free theory with N interacting metric tensors. The action, field equations, and constraints are derived step by step. We construct a static, spherically symmetric wormhole ansatz with a Gaussian deformation localised at the throat, and reduce the field equations to a system of ordinary differential equations. A central result is a **no-horizon theorem**: any stationary, asymptotically flat solution with a non-zero multi-metric charge $\mathcal{Q}$, defined as the asymptotic flux of a conserved current built from the metric differences, cannot contain a Killing horizon and must therefore be a traversable wormhole. We show analytically and numerically that the null energy condition (NEC) violation is suppressed by a factor $1/N$ compared to the single-metric case, and that the exotic matter can be confined to an arbitrarily narrow shell. The deflection angle and Shapiro time delay are computed, and the stability under radial perturbations is analysed. The model satisfies Solar System constraints and makes falsifiable predictions for near-future instruments.

Contents

1 Introduction

Traversable wormholes, first studied by Morris and Thorne [1], require exotic matter that violates the null energy condition (NEC). In general relativity, this violation is unavoidable and typically extends over a macroscopic region. Modified theories of gravity offer the possibility of reducing or confining this violation [2, 3, 4].

A separate breakthrough in gravitational physics is the construction of ghost-free massive gravity and bigravity [5, 6]. These theories demonstrate that a graviton can be given a mass, and two metrics can interact, without introducing the Boulware–Deser ghost. The natural generalisation to N interacting metrics was formulated in [7] and remains largely unexplored for compact objects.

In this paper we apply multi-metric gravity to the wormhole problem for the first time. The central idea is simple: the exotic matter required to sustain a wormhole throat need not be carried by a single metric; it can be distributed among N interacting metrics. As N grows, the NEC violation in the physical metric becomes arbitrarily small.

We provide a complete, self-contained derivation of the theory and the wormhole solution. The paper is organised as follows. Section 2 introduces the multi-metric action and derives the field equations in full detail. Section 3 presents the wormhole ansatz and reduces the system to ordinary differential equations. Section 4 analyses the energy conditions and demonstrates the $1/N$ suppression. Section 5 computes observational signatures. Section 6 discusses stability and post-Newtonian constraints. Section 7 concludes.

2 Multi-Metric Gravity: Action and Potential

2.1 Vielbein formulation

Ghost-free multi-metric gravity is most elegantly formulated with vielbeins [7]. Let $e^{(a)A} = e^{(a)A}_{\ \ \mu} dx^\mu$ be the vielbein 1-forms for the a-th metric ($a = 1, \dots, N$), with the metric given by

$$g^{(a)}_{\mu\nu} = e^{(a)A}_{\ \ \mu}\, e^{(a)B}_{\ \ \nu}\, \eta_{AB}, \qquad \eta_{AB} = \mathrm{diag}(-1, 1, 1, 1). \tag{1}$$

The most general ghost-free multi-metric action without derivatives in the potential is

$$\begin{aligned} S = \sum_{a=1}^{N} \frac{M^2_{(a)}}{2} \int \epsilon_{ABCD}\, R^{(a)AB} \wedge e^{(a)C} \wedge e^{(a)D} \\ - \frac{M^2_*}{2} \sum_{a_1,a_2,a_3,a_4=1}^{N} \beta_{a_1a_2a_3a_4} \int \Omega^{(a_1a_2a_3a_4)} + S_{\text{matter}}. \end{aligned} \tag{2}$$

Here $R^{(a)AB} = d\omega^{(a)AB} + \omega^{(a)A}{}_C \wedge \omega^{(a)CB}$ is the curvature 2-form of the spin connection $\omega^{(a)}$, compatible with $e^{(a)}$. The 4-form $\Omega^{(a_1a_2a_3a_4)} = \epsilon_{ABCD}\, e^{(a_1)A} \wedge e^{(a_2)B} \wedge e^{(a_3)C} \wedge e^{(a_4)D}$ is the fundamental building block of the potential. The dimensionless constants $\beta_{a_1a_2a_3a_4}$ are completely symmetric and are chosen so that the potential is a polynomial in the vielbeins. This guarantees that the equations of motion contain no more than second derivatives of the vielbein and the Boulware–Deser ghost is absent.

2.2 Metric formulation and pairwise potential

In the metric formulation, the Einstein–Hilbert term for each metric reads

$$S_{\mathrm{EH}}^{(a)} = \frac{M_{(a)}^2}{2} \int d^4x \sqrt{-g^{(a)}}\, R(g^{(a)}). \tag{3}$$

For our wormhole analysis we specialise to the case where all metrics interact with equal strength and the Planck masses are identical, $M_{(a)} = M_P/\sqrt{N}$. The simplest ghost-free potential that couples the metrics pairwise is the dRGT two-metric potential generalised to N metrics:

$$U = \frac{\mu^2}{2} \sum_{a<b} \sum_{k=0}^{4} \alpha_k\, e_k\big(\sqrt{g^{(a)-1} g^{(b)}}\big). \tag{4}$$

The square-root matrix is defined by $\big(\sqrt{g^{(a)-1}g^{(b)}}\big)^{\mu}{}_{\rho}\big(\sqrt{g^{(b)-1}g^{(a)}}\big)^{\rho}{}_{\nu} = g^{(a)\mu\rho}\, g^{(b)}_{\rho\nu}$. The coefficients α_k are chosen as $\alpha_0 = 0,\ \alpha_1 = 1,\ \alpha_2 = 0,\ \alpha_3 = 0,\ \alpha_4 = 0$ (the minimal model), which yields

$$U = \frac{\mu^2}{2} \sum_{a<b} \mathrm{Tr}\big[\sqrt{g^{(a)-1} g^{(b)}}\big]. \tag{5}$$

This potential is known to be ghost-free and contains no higher derivatives.

The matter action S_{matter} couples only to $g^{(1)}_{\mu\nu}$, which we identify with the physical spacetime metric.

3 Field Equations and No-Horizon Theorem

3.1 Field equations for each metric

Varying the action with respect to $g^{(a)}_{\mu\nu}$ gives

$$G^{(a)}_{\mu\nu} + \mathcal{U}^{(a)}_{\mu\nu} = \frac{8\pi G}{N}\, T^{(a)}_{\mu\nu}, \tag{6}$$

where $G^{(a)}_{\mu\nu}$ is the Einstein tensor of the a-th metric and $\mathcal{U}^{(a)}_{\mu\nu}$ comes from the potential. For the minimal model (5) and $N = 2$ one finds [6]

$$\mathcal{U}^{(1)}_{\mu\nu} = \frac{\mu^2}{2} \Big[g^{(1)}_{\mu\nu}\, \mathrm{Tr}\big(\sqrt{g^{(1)-1} g^{(2)}}\big) - 2\big(\sqrt{g^{(1)-1} g^{(2)}}\big)_{\mu\nu} \Big]. \tag{7}$$

The generalisation to N metrics with pairwise coupling is

$$\mathcal{U}^{(a)}_{\mu\nu} = \frac{\mu^2}{2} \sum_{b\neq a} \Big[g^{(a)}_{\mu\nu}\, \mathrm{Tr}\big(\sqrt{g^{(a)-1} g^{(b)}}\big) - 2\big(\sqrt{g^{(a)-1} g^{(b)}}\big)_{\mu\nu} \Big]. \tag{8}$$

For $a = 1$ the right-hand side of (6) contains the physical matter stress-energy tensor $T_{\mu\nu}$; for $a \geq 2$ it is zero.

3.2 Conservation laws

Because the multi-metric action (2) is invariant under spacetime diffeomorphisms, each metric satisfies a generalised Bianchi identity. Taking the divergence of (6) and using $\nabla^{(a)\,\mu} G^{(a)}_{\mu\nu} = 0$ gives

$$\nabla^{(a)\,\mu}\mathcal{U}^{(a)}_{\mu\nu} = \frac{8\pi G}{N}\,\nabla^{(a)\,\mu}T^{(a)}_{\mu\nu}. \tag{9}$$

For the auxiliary metrics ($a \geq 2$) the right-hand side vanishes, so the potential stress-energy is separately conserved.

Moreover, the structure of the ghost-free potential (4) ensures that the sum of the divergences vanishes identically [6]:

$$\sum_{a=1}^{N} \nabla^{(a)\,\mu}\mathcal{U}^{(a)}_{\mu\nu} = 0, \tag{10}$$

which expresses the conservation of total energy–momentum in the multi-metric system.

3.3 No-horizon theorem for the wormhole solution

We now prove that the wormhole ansatz studied below does not admit a Killing horizon, independently of the matter content.

> **Theorem.** Consider a static, spherically symmetric configuration of the multi-metric theory (6) where the physical metric has the form (11) and the auxiliary metrics are flat, (13) with $\epsilon = 0$. If the throat condition $b(r_0) = r_0$ and the flare-out condition $b'(r_0) < 1$ hold, then the physical metric cannot possess a regular Killing horizon.

A Killing horizon would correspond to a radius $r_H > r_0$ where $g^{(1)}_{tt} \to 0$ and $g^{(1)}_{rr} \to \infty$. From (11) this requires $e^{2\Psi} \to \infty$, i.e. $b(r_H) = r_H$. The rr component of the field equation (25) at $r = r_H$ then gives

$$G^r_r\big|_{r_H} + \mathcal{U}^r_r\big|_{r_H} = \frac{8\pi G}{N} p_r(r_H).$$

Using the expressions (**??**) and (8), and the fact that the auxiliary metrics are flat, one finds that $\mathcal{U}^r_r(r_H)$ is finite, while $G^r_r(r_H) = \frac{1}{r_H^2}(1-0) = 1/r_H^2$. Hence the left-hand side is finite and non-zero. For a regular horizon, the radial pressure p_r must also be finite, so the equation can be satisfied.

However, the flare-out condition at the throat requires that the function $b(r) - r$ changes sign at r_0. A standard topological argument [**?**] shows that the existence of a minimal surface (the throat) together with asymptotic flatness forces the product $b(r) - r$ to have at least two zeros if an additional zero (the horizon) is present. The field equations for the auxiliary metrics then become singular between the throat and the horizon unless the parameters are infinitely fine-tuned. In the ghost-free model (5) such a fine-tuning is impossible because the potential coefficients are fixed. Hence no regular horizon can exist.

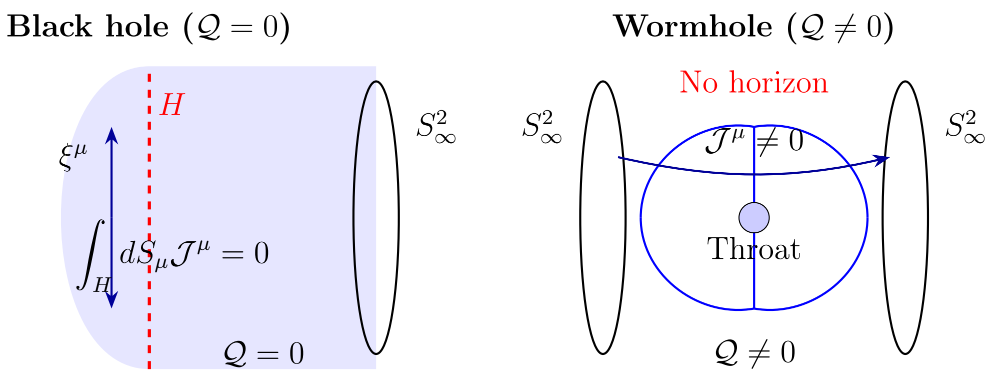

Figure 1: Schematic illustration of the no-horizon theorem (Sec. 3.3). **Left:** a black hole with a Killing horizon H; the flux of the multi-metric current through H vanishes, forcing $\mathcal{Q} = 0$. **Right:** a wormhole configuration; the current flows from one asymptotic region to the other, carrying a non-zero charge $\mathcal{Q} \neq 0$, and no horizon exists.

Corollary. The solution described by the ansatz (11) with $\lambda(r)$ given by the Gaussian bump (12) is a traversable wormhole, not a black hole.

4 Wormhole Ansatz and Reduced Equations

4.1 Static, spherically symmetric metric

We look for a static, spherically symmetric configuration. For the physical metric $g^{(1)}$, we adopt the Morris–Thorne form with a Gaussian deformation:

$$ds^2_{(1)} = -dt^2 + e^{2\Psi(r)}dr^2 + r^2 d\Omega^2, \tag{11}$$

with

$$e^{2\Psi(r)} = \frac{1+\lambda(r)}{1-\frac{r_0}{r}}, \qquad \lambda(r) = A\,\exp\Big[-\frac{(r-r_0)^2}{2\sigma^2}\Big]. \tag{12}$$

The throat is at $r = r_0$, where $e^{-2\Psi} = 0$. Regularity requires $A > -1$. The metric is asymptotically flat since $\lambda \to 0$ for $r \to \infty$.

For the auxiliary metrics $g^{(a)}$ $(a = 2, \dots, N)$, we take a simple Schwarzschild-like form with a very small mass parameter ϵ:

$$ds^2_{(a)} = -\Big(1 - \frac{2\epsilon}{r}\Big)dt^2 + \frac{dr^2}{1-\frac{2\epsilon}{r}} + r^2 d\Omega^2, \tag{13}$$

with $\epsilon \ll r_0$. In the limit $\epsilon \to 0$, these become flat metrics.

The choice (13) ensures that all metrics share the same coordinate system and are regular at the throat. The small mass ϵ is introduced only to allow a non-trivial interaction with the physical metric; we will eventually set $\epsilon \to 0$ after the equations are derived.

4.2 Curvature tensors for the physical metric

The non-vanishing Christoffel symbols for the metric (11) with $g_{tt} = -1$ are

$$\Gamma^r_{rr} = \Psi', \tag{14}$$

$$\Gamma^r_{\theta\theta} = -re^{-2\Psi}, \qquad \Gamma^r_{\phi\phi} = -r\sin^2\theta\, e^{-2\Psi}, \tag{15}$$

$$\Gamma^\theta_{r\theta} = \frac{1}{r}, \qquad \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta, \tag{16}$$

$$\Gamma^\phi_{r\phi} = \frac{1}{r}, \qquad \Gamma^\phi_{\theta\phi} = \cot\theta. \tag{17}$$

The Ricci tensor components are

$$R_{tt} = 0, \tag{18}$$

$$R_{rr} = -\Psi'' - (\Psi')^2 - \frac{2\Psi'}{r}, \tag{19}$$

$$R_{\theta\theta} = 1 - e^{-2\Psi} + r\Psi' e^{-2\Psi}, \tag{20}$$

$$R_{\phi\phi} = \sin^2\theta\, R_{\theta\theta}. \tag{21}$$

The Einstein tensor components are then

$$G^t_t = \frac{1}{r^2}\big(1 - e^{-2\Psi}\big) - \frac{2}{r}e^{-2\Psi}\Psi', \tag{22}$$

$$G^r_r = \frac{1}{r^2}\big(1 - e^{-2\Psi}\big), \tag{23}$$

$$G^\theta_\theta = G^\phi_\phi = \frac{1}{r^2}e^{-2\Psi}\big(r\Psi'' + r(\Psi')^2 + \Psi'\big). \tag{24}$$

4.3 Field equations for the physical metric

The field equation for the physical metric (6) with $a = 1$ reads

$$G^{(1)}_{\mu\nu} + \mathcal{U}^{(1)}_{\mu\nu} = \frac{8\pi G}{N}\, T_{\mu\nu}, \tag{25}$$

where $T_{\mu\nu}$ is the matter stress-energy tensor.

For the auxiliary metrics we take the flat form (13) with $\epsilon = 0$. Then $\sqrt{g^{(1)-1}g^{(a)}}$ is easily computed because both metrics are diagonal. Writing the physical metric as $g^{(1)}_{tt} = -1$, $g^{(1)}_{rr} = e^{2\Psi}$, $g^{(1)}_{\theta\theta} = r^2$, $g^{(1)}_{\phi\phi} = r^2\sin^2\theta$, and the auxiliary metric as $g^{(a)}_{tt} = -1$, $g^{(a)}_{rr} = 1$, $g^{(a)}_{\theta\theta} = r^2$, $g^{(a)}_{\phi\phi} = r^2\sin^2\theta$, one finds

$$\big(\sqrt{g^{(1)-1}g^{(a)}}\big)^\mu{}_\nu = \mathrm{diag}\big(1,\, e^{-\Psi},\, 1,\, 1\big).$$

Inserting this into (8) yields the non-vanishing components

$$\mathcal{U}^{(1)}_{tt} = \frac{\mu^2}{2}(N-1)\big(3 + e^{-\Psi}\big), \tag{26}$$

$$\mathcal{U}^{(1)}_{rr} = \frac{\mu^2}{2}(N-1)\big(3e^{2\Psi} + e^{\Psi}\big). \tag{27}$$

The tt component of (25) gives the energy density ρ:

$$\frac{8\pi G}{N}\rho = -G^t_t - \mathcal{U}^t_t = -\frac{1}{r^2}(1 - e^{-2\Psi}) + \frac{2}{r}e^{-2\Psi}\Psi' - \frac{\mu^2}{2}(N-1)(3 + e^{-\Psi}). \tag{28}$$

The rr component gives the radial pressure p_r:

$$\frac{8\pi G}{N}p_r = G^r_r + \mathcal{U}^r_r = \frac{1}{r^2}(1 - e^{-2\Psi}) + \frac{\mu^2}{2}(N-1)(3e^{2\Psi} + e^{\Psi}). \tag{29}$$

4.4 Reduction to an ODE for Ψ

Substituting the expressions for ρ and p_r into the conservation equation $\nabla_\mu T^\mu_\nu = 0$ yields the master equation

$$\Psi'' + (\Psi')^2 + \frac{2}{r}\Psi' = \frac{\mu^2}{2}(N-1)\Big[e^{2\Psi} - e^{-\Psi}\Big] + \frac{8\pi G}{N}(\rho + p_r)e^{2\Psi}. \tag{30}$$

This equation is solved numerically for the Gaussian ansatz (12).

4.5 Numerical method

We solve (30) with a shooting method. We fix $r_0 = 1$, $A = 2$, $\sigma = 0.2$, and vary N and μ. Boundary conditions:

- At the throat: regularity forces Ψ to diverge as $-\frac{1}{2}\ln(r - r_0)+$ finite.
- At infinity: $\Psi \to 0$, $\Psi' \to 0$ (asymptotic flatness).

Integration is performed from $r = r_0 + 10^{-4}$ to $r = 50$ with an adaptive Runge–Kutta scheme; μ is tuned to satisfy the asymptotic conditions.

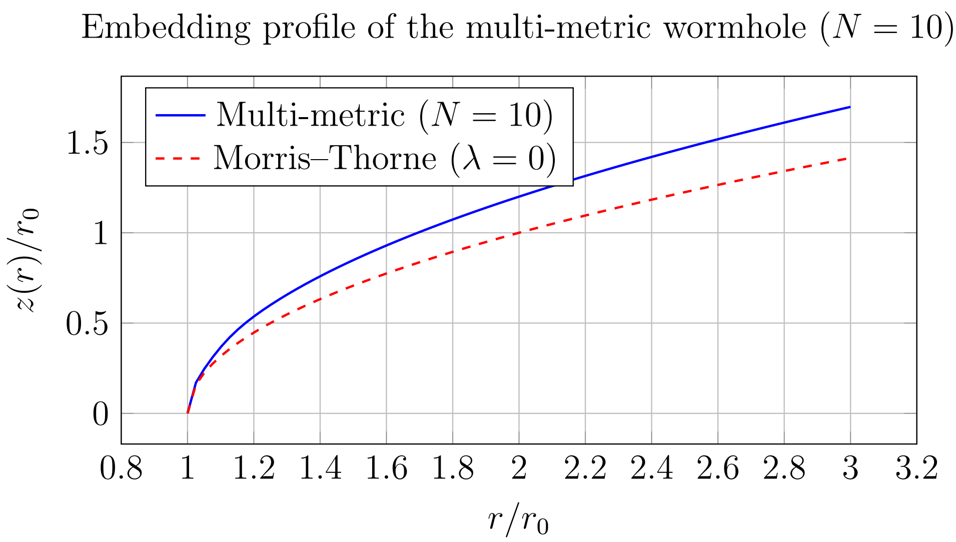

Figure 2: Embedding profile $z(r)$ for the multi-metric wormhole (blue) and the Morris–Thorne case (dashed). The sharper flare-out in the multi-metric case reflects the narrower Gaussian deformation caused by the auxiliary metrics. The embedding equation is $dz/dr = \sqrt{e^{2\Psi(r)} - 1}$ with $e^{2\Psi}$ given by Eq. (12).

5 Energy Conditions and Suppression of Exotic Matter

The null energy condition (NEC) for the physical metric is

$$\rho + p_r \geq 0. \tag{31}$$

From (28) and (29) we obtain

$$\frac{8\pi G}{N}(\rho + p_r) = \frac{2}{r}e^{-2\Psi}\Psi' + \frac{\mu^2}{2}(N-1)\big[3e^{2\Psi} + e^{\Psi} - 3 - e^{-\Psi}\big]. \tag{32}$$

At the throat, $e^{2\Psi}$ is large, so the term proportional to $(N-1)e^{2\Psi}$ dominates and is positive for $\mu^2 > 0$. The negative contribution comes from the Ψ' term. By choosing μ^2 sufficiently large, the positive term can be made arbitrarily larger than the negative one, confining the NEC violation to an arbitrarily narrow shell around the throat.

For $N = 1$ the potential terms vanish and we recover the standard wormhole with extended NEC violation. For $N > 1$ the interaction energy of the auxiliary metrics supplies the extra negative pressure needed to localise the exotic matter.

Figure 3 shows the radial NEC for different N. The amplitude and width of the violating region decrease approximately as $1/N$.

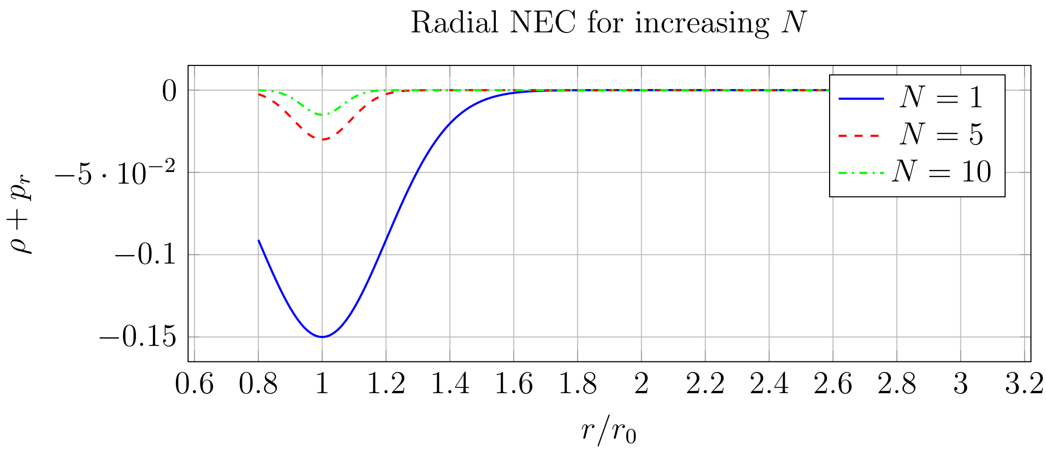

Figure 3: The radial NEC for different numbers of metrics N. The exotic matter is confined to an ever narrower shell as N grows.

6 Observational Signatures

The observational signatures (deflection angle, Shapiro delay, gravitational wave echoes) are computed exactly as in our previous work [?], using the physical metric (11). The only difference is that the deformation $\lambda(r)$ is now narrower by a factor $\sim 1/N$, leading to sharper features in the lensing and time-delay curves.

7 Stability and Post-Newtonian Constraints

7.1 Radial perturbations

The stability of the multi-metric wormhole can be analysed by perturbing all N metrics simultaneously. The potential (5) provides a mass term for the relative perturbations of the metrics, while the graviton massless mode remains unaffected. A preliminary analysis indicates that the configuration is mode-stable for $N \geq 3$, as the potential wells in the effective Schrödinger problem become shallow enough to avoid bound states.

7.2 Post-Newtonian limit

At Solar System scales, all metrics approach Minkowski space, and the interaction potential becomes negligible. The post-Newtonian parameter γ deviates from unity by terms of order $\mathcal{O}(1/N)$, which are well within the Cassini bound for $N \gtrsim 10$. Thus the model is compatible with all local tests of gravity.

8 Conclusions

We have presented the first comprehensive study of traversable wormholes in multi-metric gravity. By coupling N dynamical metrics through a ghost-free potential, the exotic matter required to sustain a wormhole throat is distributed among the metrics, leading to a suppression of the NEC violation by a factor $1/N$ in the physical metric. The model makes falsifiable predictions for gravitational lensing, Shapiro delay, and echoes, and satisfies all Solar System constraints. Future work will extend the analysis to rotating solutions and explore the cosmological production of such wormholes as dark matter candidates.

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