Finite Bernoulli–Zeta transforms for Fibonacci-type recurrence

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Abstract

Fibonacci and Lucas sequences are basic examples of second-order recurrences, and their behavior is closely connected to the golden ratio. Bernoulli numbers and special values of the Riemann zeta function also form a classical part of number theory. This paper connects these two areas through exact finite identities. The method starts from exponential generating functions, separates the odd-indexed terms, applies Bernoulli generating functions, and then compares coefficients. This gives a finite formula in which a weighted sum of zeta values at non-positive integers becomes an explicit Fibonacci expression. The same argument also gives a Lucas version, and then extends to every sequence satisfying the Fibonacci recurrence with arbitrary initial values. Exact symbolic checks and residual plot are included to show how the cancellation works. The result is a complete unconditional link between Fibonacci-type recurrences, Bernoulli numbers, and special zeta values.

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1. Introduction

The Fibonacci sequence is one of the simplest nontrivial recurrence sequences in math, yet it appears in many areas: combinatorics, continued fractions, approximation theory, algebraic number theory, discrete models, and the theory of special sequences [1]. The Lucas sequence, governed by the same recurrence but different initial values, is the natural companion sequence. Together they form a two-dimensional basis for all solutions of the Fibonacci recurrence. Their standard identities follow from the characteristic polynomial and the golden-ratio roots, but many deeper identities appear only after the sequence is placed inside a generating-function framework. A detailed account of these sequences and their applications is given by Koshy in his book [2]. Generating functions are especially effective because they convert recurrence relations into analytic identities. Ordinary generating functions reflect the algebra of the recurrence, while exponential generating functions are suited to binomial transforms, differential operators, and coefficient extraction. This paper uses exponential generating functions because the Bernoulli kernel naturally lives in that setting. The general coefficient-extraction methods used here are part of the standard finite calculus of generating functions [3].

Bernoulli numbers are defined by the generating function

$$\frac{x}{e^x - 1} = \sum_{m=0}^{\infty} B_m \frac{x^m}{m!}. \qquad (1)$$

They enter analytic number theory through the special values of the Riemann zeta function at non-positive integers. In the normalization used throughout this paper, one has

$$\zeta(1-m) = -\frac{B_m}{m}, m \geq 1. \qquad (2)$$

This formula is one of the classical points where finite arithmetic sequences meet analytic continuation. Standard treatments may be found in Apostol and Whittaker–Watson [4], [5]. Bernoulli numbers also occur naturally in the Euler–Maclaurin summation formula, where they control the correction terms connecting sums and integrals [6]. This is the same broad setting in which Ramanujan-type constants and regularized summations are often studied [7], [8]. There is an existing literature on zeta series involving Fibonacci and Lucas numbers, especially infinite series whose terms contain recurrence values and zeta values at positive arguments [9], [10].

This present work follows a different pathway. It does not evaluate infinite Fibonacci–zeta series. It proves finite identities involving recurrence sequences and zeta values at non-positive integers. This distinction is important since every identity below is a finite equality obtained from exact coefficient extraction. The central idea is to build a Bernoulli–zeta transform directly from the exponential generating function of the recurrence. The first step is a parity identity: the odd-indexed Fibonacci terms are isolated and multiplied by an exponential factor. This produces a clean expression containing the combined term $F_j + F_{2j}$. The second step inserts the Bernoulli kernels

$$\frac{x}{e^x-1}, \frac{x}{e^x+1}, \frac{x}{e^{2x}-1}, \qquad (3)$$

and extracts coefficients. The third step replaces Bernoulli numbers by zeta special values through (2). The outcome is a finite zeta identity with explicit Fibonacci weights.

The novelty of this study is the complete finite transform. It is not merely an isolated formula. The method gives the Fibonacci theorem, the Lucas theorem, and the full recurrence theorem for arbitrary initial data. This shows that the transform belongs to the structure of the recurrence itself, not to one accidental sequence.

2. Recurrence frame and Bernoulli normalization

First, let $(F_n)_{n\geq 0}$denote the Fibonacci sequence, defined by

$$F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n. \qquad (4)$$

Then, let $(L_n)_{n\geq 0}$denote the Lucas sequence, defined by

$$L_0 = 2, L_1 = 1, L_{n+2} = L_{n+1} + L_n. \qquad (5)$$

Both sequences are governed by the same characteristic equation,

$$r^2 - r - 1 = 0. \qquad (6)$$

Its roots are

$$\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}. \qquad (7)$$

These roots satisfy

$$\alpha + \beta = 1, \alpha\beta = -1, \alpha^2 = \alpha + 1, \beta^2 = \beta + 1. \qquad (8)$$

The Binet formulas are

$$F_n = \frac{\alpha^n - \beta^n}{\sqrt{5}}, L_n = \alpha^n + \beta^n. \qquad (9)$$

Define the exponential generating functions

$$\mathcal{F}(x) = \sum_{n=0}^{\infty} F_n \frac{x^n}{n!}, \mathcal{L}(x) = \sum_{n=0}^{\infty} L_n \frac{x^n}{n!}. \qquad (10)$$

Using (9), these become

$$\mathcal{F}(x) = \frac{e^{\alpha x} - e^{\beta x}}{\sqrt{5}}, \mathcal{L}(x) = e^{\alpha x} + e^{\beta x}. \qquad (11)$$

The Bernoulli numbers are normalized by (1). The first values required in the proof are

$$B_0 = 1, B_1 = -\frac{1}{2}. \qquad (12)$$

The odd Bernoulli numbers beyond B_1vanish:

$$B_{2q+1} = 0, q \geq 1. \qquad (13)$$

The zeta special-value identity (2) will be applied only for positive integer m. Thus all zeta values appearing in the final finite sums are rigorously determined by Bernoulli numbers.

we use the following convention for binomial coefficients:

$$\binom{N}{r} = 0 \text{outside the range } 0 \leq r \leq N. \qquad (14)$$

This convention allows coefficient identities to be written cleanly without exceptional endpoint cases.

3. The odd-parity exponential transform

The key structural step is the isolation of odd-indexed terms. This step is independent of Bernoulli numbers and zeta values as follows:

Lemma 1.
The Fibonacci exponential generating function satisfies

$$e^x\mathcal{F}(x) = \sum_{j=0}^{\infty} F_{2j}\frac{x^j}{j!}, \qquad (15)$$

and

$$e^x\mathcal{F}(-x) = -\mathcal{F}(x). \qquad (16)$$

Proof of Lemma 1.
Using the closed form in (11),

$$e^x\mathcal{F}(x) = \frac{e^{(\alpha+1)x} - e^{(\beta+1)x}}{\sqrt{5}}. \qquad (17)$$

By (8), $\alpha + 1 = \alpha^2$and $\beta + 1 = \beta^2$. Therefore

$$e^x\mathcal{F}(x) = \frac{e^{\alpha^2 x} - e^{\beta^2 x}}{\sqrt{5}} = \sum_{j=0}^{\infty} \frac{\alpha^{2j} - \beta^{2j}}{\sqrt{5}}\frac{x^j}{j!}. \qquad (18)$$

The coefficient in (18) is F_{2j}, which proves (15).

For the second identity,

$$e^x\mathcal{F}(-x) = \frac{e^{(1-\alpha)x} - e^{(1-\beta)x}}{\sqrt{5}}. \qquad (19)$$

Since $1 - \alpha = \beta$and $1 - \beta = \alpha$, this becomes

$$e^x\mathcal{F}(-x) = \frac{e^{\beta x} - e^{\alpha x}}{\sqrt{5}} = -\mathcal{F}(x). \qquad (20)$$

The lemma follows.

Theorem 2.
For every complex x,

$$2e^x \sum_{m=0}^{\infty} F_{2m+1}\frac{x^{2m+1}}{(2m+1)!} = \sum_{j=1}^{\infty}\left(F_j + F_{2j}\right)\frac{x^j}{j!}. \qquad (21)$$

Proof of Theorem 2.
The odd part of $\mathcal{F}$is obtained by subtracting its value at $-x$:

$$\mathcal{F}(x) - \mathcal{F}(-x) = 2\sum_{m=0}^{\infty} F_{2m+1}\frac{x^{2m+1}}{(2m+1)!}. \qquad (22)$$

Multiplication by e^x, followed by Lemma 1, gives

$$e^x\mathcal{F}(x) - e^x\mathcal{F}(-x) = \sum_{j=0}^{\infty} F_{2j}\frac{x^j}{j!} + \sum_{j=0}^{\infty} F_j\frac{x^j}{j!}. \qquad (23)$$

The constant term on the right is $F_0 + F_0 = 0$, so the series begins at $j = 1$. This proves (21).

Theorem 2 is the point where the recurrence structure becomes ready for Bernoulli extraction. The term $F_j + F_{2j}$is not inserted artificially; it is forced by the exponential shift $x \mapsto e^x\mathcal{F}(x)$.

4. Bernoulli coefficient calculus

The Bernoulli transform begins with the elementary identity

$$\frac{x}{e^x - 1} + \frac{x}{e^x + 1} = \frac{2xe^x}{e^{2x} - 1}. \qquad (24)$$

Multiplying both sides of Theorem 2 by $x/(e^{2x} - 1)$and using (24) gives

$$\left(\frac{x}{e^x - 1} + \frac{x}{e^x + 1}\right)\sum_{m=0}^{\infty} F_{2m+1}\frac{x^{2m+1}}{(2m+1)!} = \frac{x}{e^{2x} - 1}\sum_{j=1}^{\infty}\left(F_j + F_{2j}\right)\frac{x^j}{j!}. \qquad (25)$$

The three Bernoulli expansions required for coefficient extraction are now fixed. From (1),

$$\frac{x}{e^x + 1} = \sum_{r=0}^{\infty}(1 - 2^r)B_r\frac{x^r}{r!}. \qquad (26)$$

Adding (1) and (26) gives

$$\frac{x}{e^x - 1} + \frac{x}{e^x + 1} = \sum_{r=0}^{\infty}(2 - 2^r)B_r\frac{x^r}{r!}. \qquad (27)$$

Replacing $x$by $2x$in (1), then dividing by 2, gives

$$\frac{x}{e^{2x} - 1} = \sum_{r=0}^{\infty} 2^{r-1}B_r\frac{x^r}{r!}. \qquad (28)$$

Proposition 3.
For every integer $N \geq 1$,

$$\sum_{m=0}^{\lfloor (N-1)/2 \rfloor} \binom{N}{2m+1} F_{2m+1}(2-2^{N-2m-1})B_{N-2m-1} = \sum_{n=0}^{N-1} \binom{N}{n+1} (F_{n+1}+F_{2n+2})2^{N-n-2}B_{N-n-1}. \qquad (29)$$

Proof of Proposition 3.
Insert (27) and (28) into (25). The coefficient of $x^N/N!$ on the left side is obtained by choosing x^{N-2m-1} from the Bernoulli series and x^{2m+1} from the odd Fibonacci series. The factorials combine as

$$\frac{N!}{(N-2m-1)!\,(2m+1)!} = \binom{N}{2m+1}. \qquad (30)$$

This gives the left side of (29).

On the right side of (25), the coefficient of $x^N/N!$ is obtained by choosing x^{N-n-1} from the Bernoulli series in (28) and x^{n+1} from the recurrence series. The factorials give

$$\frac{N!}{(N-n-1)!\,(n+1)!} = \binom{N}{n+1}. \qquad (31)$$

The remaining Bernoulli factor is $2^{N-n-2}B_{N-n-1}$ which gives the right side of (29). Since both coefficients come from the same identity (25), the proposition is proved.

Proposition 3 is the finite Bernoulli identity from which the zeta identities follow. No convergence issue remains after coefficient extraction, because each coefficient contains only finitely many terms.

5. The main Fibonacci–zeta identity

The even specialization of Proposition 3 is decisive since of the vanishing pattern of odd Bernoulli numbers.

Theorem 4.
For every integer $k \geq 2$,

$$2k \sum_{n=0}^{2k-3} \binom{2k-1}{n+1} (F_{n+1}+F_{2n+2})2^{2k-n-2}\zeta(n+2-2k) = \frac{F_{4k}+F_{2k}}{2} - k(F_{4k-2}+F_{2k-1}). \qquad (32)$$

Proof of Theorem 4.
Set $N = 2k$ in Proposition 3. The Bernoulli index on the left side of (29) is $2k-2m-1$, always odd. For $0 \leq m \leq k-2$, this index is an odd integer greater than one, so the corresponding Bernoulli number vanishes by (13). The remaining endpoint $m = k-1$ has Bernoulli index one, but the multiplier $2-2^1$is zero. Hence the whole left side of (29) is zero.

Therefore the right side of (29) satisfies

$$\sum_{n=0}^{2k-1} \binom{2k}{n+1} (F_{n+1}+F_{2n+2}) 2^{2k-n-2} B_{2k-n-1} = 0. \qquad (33)$$

Then separate the two endpoint terms $n = 2k - 2$and $n = 2k - 1$. The first contains B_1, and the second contains B_0. Using (12), one obtains

$$\sum_{n=0}^{2k-3} \binom{2k}{n+1} (F_{n+1}+F_{2n+2}) 2^{2k-n-2} B_{2k-n-1} = k(F_{2k-1}+F_{4k-2}) - \frac{F_{2k}+F_{4k}}{2}. \qquad (34)$$

For every n in the remaining sum, the Bernoulli index $2k - n - 1$ is at least two. Formula (2) gives

$$B_{2k-n-1} = -(2k-n-1)\zeta(n + 2 - 2k). \qquad (35)$$

The binomial simplification

$$\binom{2k}{n+1} (2k-n-1) = 2k \binom{2k-1}{n+1} \qquad (36)$$

turns (34) into (32).

Theorem 4 is the main result of this paper. It is an exact finite identity. The zeta arguments in the sum are

$$2 - 2k,\ 3 - 2k,\ \ldots,\ -1, \qquad (37)$$

so every zeta value involved is a special value at a non-positive integer.

Example 1.
For $k = 2$, Theorem 4 gives

$$4 \sum_{n=0}^{1} \binom{3}{n+1} (F_{n+1}+F_{2n+2}) 2^{2-n} \zeta(n-2) = \frac{F_8 + F_4}{2} - 2(F_6 + F_3). \qquad (38)$$

Using $\zeta(-2) = 0$, $\zeta(-1) = -1/12$, and the Fibonacci values

$$F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_6 = 8, F_8 = 21, \qquad (39)$$

both sides equal -8. This confirms the first nontrivial case exactly.

6. Lucas and general recurrence extensions

The transform belongs to the recurrence structure, not only to the Fibonacci normalization. The Lucas identity follows from the same parity method.

Theorem 5.
For every integer $k \geq 2$,

$$2k \sum_{n=0}^{2k-3} \binom{2k-1}{n+1} (L_{2n+2} - L_{n+1}) 2^{2k-n-2} \zeta(n+2-2k) = \frac{L_{4k} - L_{2k}}{2} - k(L_{4k-2} - L_{2k-1}). \qquad (40)$$

Proof of Theorem 5.
The odd part of the Lucas exponential generating function is

$$\mathcal{L}(x) - \mathcal{L}(-x) = 2 \sum_{m=0}^{\infty} L_{2m+1} \frac{x^{2m+1}}{(2m+1)!}. \qquad (41)$$

Using (11),

$$e^x \mathcal{L}(x) = e^{\alpha^2 x} + e^{\beta^2 x} = \sum_{j=0}^{\infty} L_{2j} \frac{x^j}{j!}, \qquad (42)$$

and

$$e^x \mathcal{L}(-x) = e^{\beta x} + e^{\alpha x} = \mathcal{L}(x). \qquad (43)$$

Thus

$$2e^x \sum_{m=0}^{\infty} L_{2m+1} \frac{x^{2m+1}}{(2m+1)!} = \sum_{j=1}^{\infty} (L_{2j} - L_j) \frac{x^j}{j!}. \qquad (44)$$

Repeating the Bernoulli coefficient extraction from Section 4 with $L_{2j} - L_j$in place of $F_j + F_{2j}$gives

$$\sum_{n=0}^{2k-1} \binom{2k}{n+1} (L_{2n+2} - L_{n+1}) 2^{2k-n-2} B_{2k-n-1} = 0. \qquad (45)$$

Separating the endpoint terms gives

$$\sum_{n=0}^{2k-3} \binom{2k}{n+1} (L_{2n+2} - L_{n+1}) 2^{2k-n-2} B_{2k-n-1} = k(L_{4k-2} - L_{2k-1}) - \frac{L_{4k} - L_{2k}}{2}. \qquad (46)$$

Equations (35) and (36) convert this Bernoulli identity into (40).

The Fibonacci and Lucas sequences form a basis for the full solution space of the recurrence. This gives the general version immediately.

Corollary 6.
Let $(U_n)_{n\geq 0}$ be any sequence satisfying

$$U_{n+2} = U_{n+1} + U_n, \qquad (47)$$

with arbitrary complex initial values U_0 and U_1. Put

$$a = U_1 - \frac{U_0}{2}, b = \frac{U_0}{2}. \qquad (48)$$

Then $U_n = aF_n + bL_n$ for every $n \geq 0$. Consequently, for every integer $k \geq 2$,

$$2k \sum_{n=0}^{2k-3} \binom{2k-1}{n+1} [a(F_{n+1}+F_{2n+2})+b(L_{2n+2}-L_{n+1})]2^{2k-n-2}\zeta(n+2-2k)$$
$$= a\left[\frac{F_{4k}+F_{2k}}{2} - k(F_{4k-2}+F_{2k-1})\right] + b\left[\frac{L_{4k}-L_{2k}}{2} - k(L_{4k-2}-L_{2k-1})\right]. \qquad (49)$$

Proof of Corollary 6.
At $n = 0$ and $n = 1$, the sequence $aF_n + bL_n$ has the values

$$aF_0 + bL_0 = U_0, aF_1 + bL_1 = U_1. \qquad (50)$$

Both U_n and $aF_n + bL_n$ satisfy (47), so equality holds for all n. Multiplying Theorem 4 by a, multiplying Theorem 5 by b, and adding the two identities gives (49).

Corollary 6 completes the recurrence extension. The transform applies to the entire family of Fibonacci-type sequences, not only to the two canonical bases.

The results are proved algebraically, but exact computation is a useful way to inspect the cancellation. Define the Fibonacci residual by

$$R_F(k) = 2k \sum_{n=0}^{2k-3} \binom{2k-1}{n+1} (F_{n+1}+F_{2n+2})2^{2k-n-2}\zeta(n+2-2k) - \frac{F_{4k}+F_{2k}}{2} + k(F_{4k-2}+F_{2k-1}). \qquad (51)$$

Define the Lucas residual by

$$R_L(k) = 2k \sum_{n=0}^{2k-3} \binom{2k-1}{n+1} (L_{2n+2}-L_{n+1})2^{2k-n-2}\zeta(n+2-2k) - \frac{L_{4k}-L_{2k}}{2} + k(L_{4k-2}-L_{2k-1}). \qquad (52)$$

By Theorems 4 and 5, the residuals vanish identically:

$$R_F(k) = 0, R_L(k) = 0, k \geq 2. \qquad (53)$$

This means that $R_F(k)$ and $R_L(k)$ record the exact difference between each finite Bernoulli–zeta sum and its corresponding closed recurrence expression. Their value is zero because the two main transform identities have already proved that the finite sums close exactly.

Figure 1 shows this cancellation process for the representative case $k = 24$. For clarity, write the Fibonacci closed value as

$$C_k^{(F)} = \frac{F_{4k} + F_{2k}}{2} - k(F_{4k-2} + F_{2k-1}), \qquad (54)$$

and the Lucas closed value as

$$C_k^{(L)} = \frac{L_{4k} - L_{2k}}{2} - k(L_{4k-2} - L_{2k-1}). \qquad (55)$$

The two curves in the plot are the running partial sums of the Fibonacci and Lucas transforms after normalization by $C_k^{(F)}$and $C_k^{(L)}$, respectively. Each marker therefore represents an exact partial sum, not an approximation to the theorem.

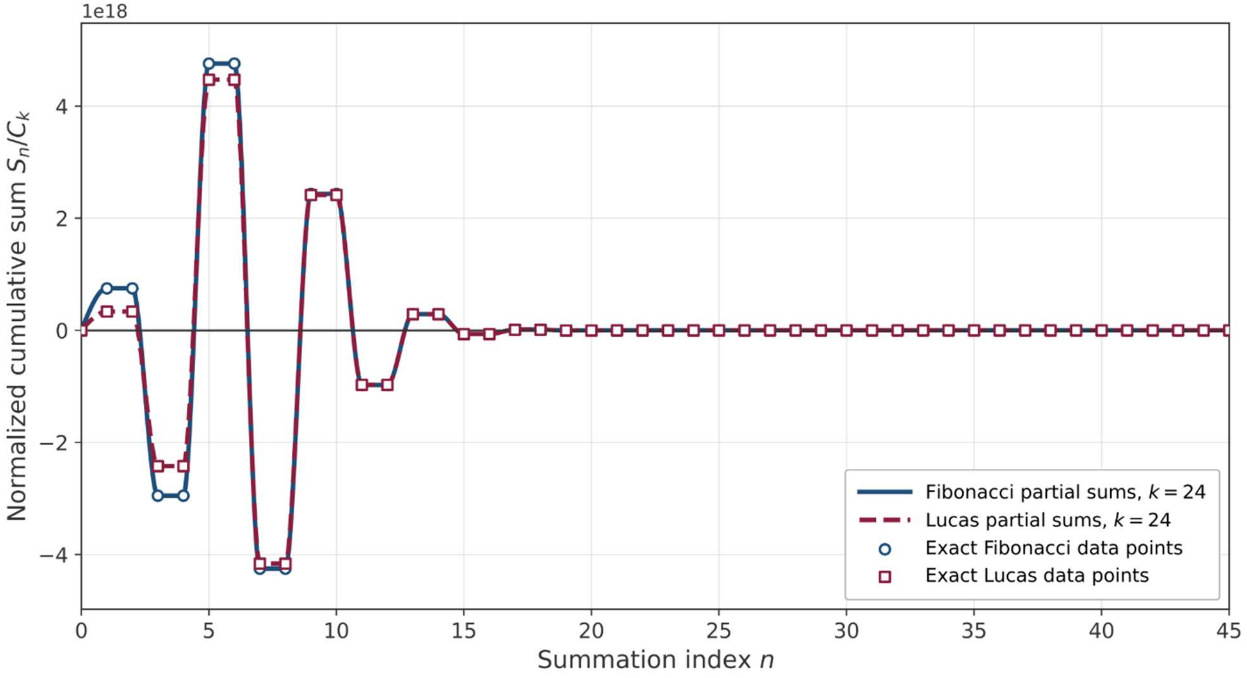

Figure 1. Normalized cumulative cancellation in the finite Bernoulli–zeta transform

The graph makes visible what the formulas prove algebraically. At $k = 24$, the early zeta-weighted terms are large, with magnitudes reaching the scale of 10^{18}. The final identity is therefore comes from a precise cancellation among large positive and negative contributions. After all terms have been included, the normalized value is exactly 1, showing that the finite Bernoulli–zeta sum has recovered the closed Fibonacci or Lucas expression completely.

7. Conclusion

In this study, we established a finite Bernoulli–zeta transform for Fibonacci-type recurrence sequences. The construction begins with the exponential generating functions of the Fibonacci and Lucas sequences, separates the odd-indexed terms, and then applies Bernoulli generating kernels. Exact coefficient extraction gives a finite Bernoulli identity, and the classical relation between Bernoulli numbers and zeta values converts it into a finite identity involving the Riemann zeta function at non-positive integers.

The main result proves that a weighted finite sum of zeta special values can be written exactly as a closed Fibonacci expression. The same method gives a Lucas companion formula. Since the Fibonacci and Lucas sequences form a basis for all sequences satisfying the recurrence:

$$U_{n+2} = U_{n+1} + U_n,$$

the transform extends naturally to every sequence of this type with arbitrary initial values.

The important point is that the identities are finite and unconditional. No limiting process, convergence argument, asymptotic estimate, or unproved assumption is used. Each equality follows from explicit generating-function algebra and exact coefficient comparison. The residual diagnostics and the normalized cancellation figure also illustrate the same fact numerically which large positive and negative Bernoulli–zeta contributions cancel exactly and recover the closed recurrence value.

Dedication

To my professors, whose teaching gave me the foundation to think carefully, write clearly, and respect the beauty of a well-built proof. I am deeply grateful for the patience, encouragement, and mathematical insight you shared with me

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