

# Quantum Oscillator Lattice: A Unified Origin of Fundamental Constants and Fields

Juan Moreno Borrillo

(Dated: July 29, 2025)

**Abstract.** *This paper presents the foundational premises for a unified physical theory in which the vacuum is modeled as an elastic lattice of coupled quantum harmonic oscillators. Rather than aiming to construct a complete unified theory, our goal is to explore the consequences of a set of physically motivated assumptions and to demonstrate their consistency with established theoretical principles and dimensional relations. Within this framework, gravitational, electromagnetic, and thermo-entropic interactions are interpreted as distinct geometric deformation modes of a single symmetric field tensor  $\mathcal{G}_{\mu\nu}$ , and fundamental constants of nature emerge naturally from the oscillatory dynamics. We show that these assumptions lead to coherent interpretations of field sources (mass, charge, temperature) sharing a common oscillatory origin in spacetime. While non-exhaustive, our treatment lays a physically and mathematically grounded path for future development of a unified field theory anchored in quantum-elastic principles.*

*”Entia non sunt multiplicanda praeter necessitatem”*

— Ockham’s Razor

*”Padre, Señor del cielo y de la tierra, te doy gracias porque has ocultado todo esto a los sabios y entendidos y se lo has revelado a los que son como niños.”*

— Matthew 11:25

## I. INTRODUCTION

The quest for a unified theoretical framework capable of describing all fundamental interactions from a common origin remains a central theme in contemporary physics [1]. Despite the tremendous success of the Standard Model of particle physics in unifying electromagnetic, weak and strong forces [2], and of General Relativity (GR) in geometrizing gravity [3, 4], a conceptual schism persists between the quantum field theories (QFTs) of the former and the geometric description of the latter [5]. Moreover, observational puzzles such as dark energy and dark matter [6–8], together with the inability to quantize gravity in a conventional QFT framework [9], underscore the need for a deeper structure underlying both regimes.

Thermodynamic and emergent-gravity approaches have hinted at such a structure. Jacobson’s derivation of Einstein’s equations from local entropy balance [10], Verlinde’s entropic gravity proposal [11], and the striking analogies between black-hole thermodynamics and vacuum fluctuations [12] point toward an intimate link between entropy, quantum vacuum dynamics, and spacetime geometry, and suggests that disparate forces may be just different manifestations of a single underlying field.

In this work, *we explore the hypothesis that the quantum vacuum itself—spacetime at its most*

*fundamental level—is not a passive background but a dynamic, elastic medium.* We build upon the foundational principle of Quantum Field Theory, where free fields are mathematically described as an infinite collection of quantum harmonic oscillators [13]. We elevate this description from a mathematical tool to a physical model, postulating that the vacuum is fundamentally a *quantum oscillator lattice*. This lattice structure introduces a natural discreteness at the most fundamental scale, aligning our model with approaches such as Loop Quantum Gravity [14] and Causal Set Theory [15]. The collective state of this oscillatory medium is described by a single, symmetric rank-2 tensor field,  $\mathcal{G}_{\mu\nu}(x)$ , which represents the local strain or deformation of the vacuum. All observed particles and force fields become emergent, large-scale manifestations of the different vibrational, shear, or torsional modes of this underlying lattice.

Within this framework, electromagnetic, gravitational and thermodynamic phenomena emerge as different vibrational, shear or torsional modes of the same quantum harmonic oscillator’s lattice. *Fundamental constants of nature lose their “arbitrary” character and become the metric coefficients of a generalized physical geometry—effectively the “metric” that converts one nominal unit into another.* This unification provides a self-consistency check, as setting the SI units as equivalent at the fundamental level (via the constants) leads to a coherent, contradiction-free description of nature.

An important clarification is that we do not attempt to construct a full and complete unified theory. Rather, we propose and explore a set of physically motivated postulates, and analyze how far these assumptions can go in explaining the emergence of known fields and constants, and in revealing coherent dimensional structures. The aim of this paper is to evaluate the internal consistency, physical plausibility, and predictive coherence of these assumptions—not to deliver a fully quantized or dynamically complete theory. In this sense, the present work should be seen as a conceptual and dimensional groundwork for future unified field models.

## Part I: The Unified Dimensional Framework

### II. THE UNIFIED FIELD LAGRANGIAN AND THE DIMENSIONAL COLLAPSE INTO SPACE-TIME EQUIVALENCES

#### A. Derivation of Mass-Inductance Equivalence from a Unified Field Lagrangian

Let us construct the most general, second-order Lorentz-invariant Lagrangian for the dynamics of a unified field  $\mathcal{G}_{\mu\nu}$ . The Lagrangian must contain a kinetic term encoding the inertial response of the field and a potential term  $V(\mathcal{G}_{\mu\nu})$  encoding its elastic and mass-like properties. The kinetic term takes the form:

$$\mathcal{L}_{kinetic} = \frac{1}{2}\kappa(\partial^\alpha\mathcal{G}^{\mu\nu})(\partial_\alpha\mathcal{G}_{\mu\nu}) \quad (1)$$

where  $\kappa$  is a single, fundamental constant representing the intrinsic stiffness or inertial resistance of the vacuum substrate. The unity of the field requires a single such constant.

The total kinetic energy of the field, associated with its time evolution, is given by the temporal component ( $\alpha = 0$ ) of this term:

$$\mathcal{T}_{kin} = \frac{1}{2}\kappa(\partial_0\mathcal{G}^{\mu\nu})(\partial_0\mathcal{G}_{\mu\nu}) \quad (2)$$

To connect this abstract formalism to known physics, we make the following physically-motivated identifications for the primary deformation modes. Scalar (compressional) deformations are naturally associated with mass-like sources, while torsional (shear) deformations are associated with the flow of sources (currents):

- The component  $\mathcal{G}_{00}$  encodes scalar deformations, associated with mass-like gravitational sources.

- The components  $\mathcal{G}_{0i}$  encode torsional (azimuthal) modes, associated with currents and thus analogous to magnetic fields.

These identifications will be further expanded and justified in Section XXII B.

Expanding the kinetic energy term reveals the distinct contributions:

$$\mathcal{T}_{kin} = \frac{1}{2}\kappa [(\partial_0\mathcal{G}_{00})^2 + 2(\partial_0\mathcal{G}_{0i})^2 + (\partial_0\mathcal{G}_{ij})^2 + \dots] \quad (3)$$

We can now directly apply the oscillator analogies central to this paper:

1. **Mass-like Inertia:** The kinetic energy of the scalar mode,  $\frac{1}{2}\kappa(\partial_0\mathcal{G}_{00})^2$ , is the field-theoretic analogue of the mechanical kinetic energy  $\frac{1}{2}m\dot{x}^2$ . In this context, the coefficient  $\kappa$  has the physical dimension of **Mass**.
2. **Inductance-like Inertia:** The kinetic energy of the torsional modes,  $\kappa(\partial_0\mathcal{G}_{0i})^2$ , is the analogue of the energy stored in an inductor,  $\frac{1}{2}L\dot{q}^2$ . Here,  $\dot{q}$  corresponds to the "velocity" of the field excitation sourced by a current. Thus, the same coefficient  $\kappa$  must have the physical dimension of **Inductance**.

Since Lorentz covariance and the principle of a single unified field demand that a single constant  $\kappa$  governs the inertial properties of all components of  $\mathcal{G}_{\mu\nu}$ , it follows that the dimensional character of mass and inductance must be identical.

$$\boxed{[\text{Mass}] \equiv [\kappa] \equiv [\text{Inductance}]} \quad (4)$$

The above equivalence (from now on, the Inertial Equivalence Principle) becomes a direct and necessary consequence of describing a unified reality with a single, covariant field Lagrangian. This equivalence does not imply that mass and inductance are equal in any conventional system, but rather that they play *isomorphic roles* within the elastic vacuum model: both represent resistance to changes in oscillatory motion—mechanical or electromagnetic—and encode the same form of reactive inertia under field excitation. Thus, this Principle is not arbitrary, but a natural requirement of modal unification, guiding the redefinition of all fundamental units from first principles.

#### *Dimensional consequences of the Inertial Equivalence Principle*

Taking the SI units of inductance  $L$  as  $[ML^2I^{-2}T^{-2}]$ , the Inertial Equivalence Principle leads directly to:

$$[M] \equiv [ML^2I^{-2}T^{-2}],$$

For dimensional consistency, this last equivalence requires that the combination  $[L^2I^{-2}T^{-2}]$  must be dimensionless. Solving for the dimension of current  $[I]$  yields:

$$[I^2] \equiv [L^2T^{-2}] \implies \boxed{[I] \equiv [LT^{-1}]} \quad (5)$$

This result, stemming directly from the Inertial Equivalence Principle, implies that electric current within this unified picture acquires the dimensions of velocity. The consequences of this dimensional assignment will be explored throughout this work.

As a sanity check, the resistance  $R$  in an RLC circuit is analogous to the damping coefficient  $b$  in a mechanical oscillator. Establishing the dimensional equivalence between them, we find that:

$$[MT^{-1}] \equiv [ML^2T^{-3}I^{-2}],$$

which implies that  $[L^2I^{-2}T^{-2}]$  becomes dimensionless, as we had obtained just before, and which leads to the dimensionality found for  $I$ .

### B. The Kinematic Nature and Geometric Interpretation of Current

The result derived in the preceding subsection,  $[I] \equiv [LT^{-1}]$ , is a non-trivial consequence of the Inertial Equivalence Principle. It establishes a fundamental connection between electrodynamics and kinematics. In this section, we explore the profound physical implications of this dimensional equivalence.

#### *Theoretical Simplification and a Unified Dimensional System*

In the conventional SI framework, the Ampere is treated as a base unit, making current dimensionally independent of mass, length, and time. While this is operationally effective, it can obscure underlying physical connections. Our derivation reveals that the dimension of current is, ultimately, composed of the dimensions of length and time.

This does not alter any measurable physical law or experimental outcome. Rather, it simplifies the theoretical landscape by reducing the number of fundamental dimensions required to describe nature. Physical quantities such as impedance, permittivity, and field strength, which have complex dimensions in the SI system, can now be expressed more simply within a unified spacetime geometry.

#### *Current as the Velocity of a Propagating Deformation*

The equivalence  $[I] \equiv [LT^{-1}]$  provides a powerful physical insight into the nature of current within our elastic vacuum model. By definition, current is the rate of flow of charge,  $I = dQ/dt$ . If charge, like mass, is understood as a localized deformation of the vacuum field  $\mathcal{G}_{\mu\nu}$ , then current is necessarily the *propagation velocity of that deformation wave*.

This picture moves beyond the classical image of charge particles moving *through* space. Instead, current *is* the propagation of a specific mode of the vacuum's deformation. As identified in our analysis of the kinetic energy term, currents are associated with the torsional (shear) modes of the field,  $\mathcal{G}_{0i}$ . Therefore, we interpret an electric current as the velocity of a torsional wave propagating through the elastic spacetime lattice.

This interpretation also provides a natural bridge between the source terms in gravitational and electromagnetic equations. Both mass (sourcing scalar deformation,  $\mathcal{G}_{00}$ ) and charge (sourcing torsional deformation via its flow) alter the local geometry of the vacuum. Their respective currents—mass flow and electric current—can now be seen as conjugate expressions of the same underlying geometrodynamics principle: the propagation of deformations in the unified field.

This dimensional identification sets the stage for a unified treatment of source terms across field equations, and invites a reinterpretation of current as a *dynamical field generator*, not merely a charge derivative. Redefining the ampere as velocity not only simplifies the dimensional landscape, but also aligns electromagnetism with the spacetime symmetries that govern motion and gravity, contributing to the internal coherence of the unified field model.

### C. Dimensional Collapse and Space-Time Equivalence in the Unified Field

#### *Derivation of the Structural Equivalence of Coupling Constants*

In modern field theory, interactions are dictated by gauge symmetries [16], and the sources of the fields are the conserved currents associated with these symmetries by Noether's Theorem. In a unified field, the full Lagrangian  $\mathcal{L}(\mathcal{G}_{\mu\nu})$  must be invariant under a unified gauge group, which we will denominate  $\mathbb{G}_U$ , acknowledging that:

1. **Source of Gravity:** The source of

the gravitational field is the stress-energy-momentum (SEM) tensor,  $T^{\mu\nu}$ . As demonstrated within formalisms like the UG theory [17] (XXVIB),  $T^{\mu\nu}$  is the conserved Noether current corresponding to *spacetime translation symmetry*. The coupling constant governing this interaction is the gravitational constant,  $G$ .

2. **Source of Electromagnetism:** The source of the electromagnetic field is the four-current,  $J_{em}^\mu$ . This is the conserved Noether current corresponding to the internal  $U(1)$  *gauge symmetry*. The coupling constant governing this interaction is, effectively, the Coulomb constant,  $K_e = 1/(4\pi\epsilon_0)$ .

In a truly unified theory, both spacetime symmetries and internal gauge symmetries must be different manifestations of the single, overarching gauge group  $\mathbb{G}_U$ . Gravity and electromagnetism are not separate interactions, but different "charges" (conserved currents) to which the unified field  $\mathcal{G}_{\mu\nu}$  couples. Therefore, the coupling constants  $G$  and  $K_e$  must not be fundamentally different types of parameters. They both quantify the universal response of the vacuum substrate to different types of conserved source currents. In a unified framework where these sources are governed by a single gauge principle, their respective coupling constants must share the same dimensional character. As a result,  $[G] \equiv [K_e]$  (the Coupling Equivalence Principle) becomes a requirement for a consistent unification of the sources of interaction under a single gauge structure.

$$\boxed{[G] \equiv [K_e]} \quad (6)$$

*Dimensional consequences of the Coupling  
Equivalence Principle*

Let us rigorously derive the consequences of this principle combined with our previous findings. The conventional SI dimensions are:

$$[G] = [M^{-1}L^3T^{-2}], \quad [K_e] = [ML^3T^{-4}I^{-2}].$$

Setting  $[G] \equiv [K_e]$ , we obtain:

$$[M^{-1}L^3T^{-2}] \equiv [ML^3T^{-4}I^{-2}].$$

Rearranging for  $[M]$ , and recalling that within the unified framework we have established  $[I] \equiv [LT^{-1}]$  (5), we substitute  $I^{-2}$  as:

$$[M^{-1}L^3T^{-2}] \equiv [ML^3T^{-4}(L^{-2}T^2)].$$

Simplifying, we find:

$$\begin{aligned} [M^{-1}L^3T^{-2}] &\equiv [MLT^{-2}] \rightarrow \\ [M^{-1}L^3] &\equiv [ML] \rightarrow \\ [L^2] &\equiv [M^2] \rightarrow \\ \boxed{[M] \equiv [L]} & \end{aligned} \quad (7)$$

This outcome, contingent on Inertial Equivalence Principle 4 and the Coupling Equivalence Principle 6, signifies that mass and length share the same fundamental dimension within this theoretical structure. From this result and the previous ones, we can substitute  $[M]$  and  $[I]$  in the previous equivalence  $[MT^{-1}] \equiv [ML^2T^{-3}I^{-2}]$ , to get that  $[T^{-4}L^4]$  becomes dimensionless; which, in turn, implies that we have reached the fundamental equivalence

$$\boxed{[M] \equiv [L] \equiv [T]} \quad (8)$$

This result establishes a profound conclusion: *mass, length and time are fundamentally equivalent in the common unified field* (from now on, referred to as Spacetime Equivalence Principle). This equivalence leads to a natural collapse of dimensions -as we will see-, implying that the evolution of the universe should be understood in terms of oscillatory interactions where mass-energy, length and time emerge as manifestations of a unified, underlying geometric structure.

### III. THE UNIFIED DIMENSIONAL FRAMEWORK

The Spacetime Equivalence Principle ( $[M] \equiv [L] \equiv [T]$ ) implies a radical simplification and unification of the dimensional structure of physics. In this section, we establish the dimensions of all relevant physical quantities in terms of a single geometric unit,  $[L]$ .

#### A. Geometrization of Physical Sources

The principle's most immediate consequence is that the fundamental sources of physical interactions—mass, charge, and thermal energy—are revealed to be dimensionally equivalent expressions of spacetime geometry.

- **Mass:** By direct application of the principle,  $[M] \equiv [L]$ .
- **Energy:** From the mass-energy equivalence,  $E = mc^2$ , and as velocity,  $c$ , is dimensionless in this framework, the dimension of energy becomes identical to that of mass. Thus,  $[E] \equiv [M] \equiv [L]$ .

- **Charge:** Starting with the definition  $[Q] = [I \cdot T]$  and applying  $[I] \equiv [1]$  and the Space-time Equivalence Principle ( $[T] \equiv [L]$ ), we find  $[Q] \equiv [1 \cdot L] \equiv [L]$ .
- **Temperature:** As  $k_B T$  has dimensions of energy, and we will show that  $k_B$  becomes dimensionless (VC), it has the same dimensions of energy. Thus,  $[T_{\text{emp}}] \equiv [E] \equiv [L]$ .

## B. Dimensionless Nature of Dynamical and Coupling Constants

The constants governing dynamics and coupling strengths become dimensionless pure numbers within the unified framework.

### *Dynamical Quantities*

- **Velocity and Current:** We have that  $[v] = [LT^{-1}] \equiv [1]$  and  $[I] \equiv [1]$  (5).
- **Resistance:** From the equivalence with mechanical damping,  $[R] \equiv [MT^{-1}]$ . Substituting the base equivalences gives  $[L \cdot L^{-1}] \equiv [1]$ .
- **Voltage and Power:** From Ohm's Law,  $[V] = [I \cdot R] \equiv [1 \cdot 1] \equiv [1]$ . Consequently, Power  $[P] = [V \cdot I] \equiv [1]$ .

### *Fundamental Coupling Constants*

- **Permittivity and Permeability:** The electric permittivity  $[\epsilon_0] = [M^{-1}L^{-3}T^4I^2]$  becomes  $[L^{-1}L^{-3}L^4 \cdot 1^2] \equiv [1]$ . Consequently, the magnetic permeability  $[\mu_0] = [1/\epsilon_0 c^2]$  is also dimensionless.
- **$G$  and  $K_e$ :** The gravitational constant  $[G] = [M^{-1}L^3T^{-2}]$  becomes  $[L^{-1}L^3L^{-2}] \equiv [1]$ . As required by the Coupling Equivalence Principle, Coulomb's constant  $[K_e]$  also becomes dimensionless.
- **Boltzmann's constant  $k_B$ :** From the equivalence derived in (VC), it becomes dimensionless.
- **The fine-structure constant  $\alpha$ :** By its definition,  $\alpha = \frac{e^2}{2\epsilon_0 \hbar c}$ . One can check that using the previous dimensions described, it is dimensionless, as expected.

The dimensionless nature of these constants is a strong indicator of a deeply unified structure, where the laws of physics are ultimately described by pure geometry and topology.

As a separate category, **Planck's constant**,  $h$ , the quantum of action, has dimensions of  $[E \cdot T]$ . In our framework, this becomes  $[L \cdot L] = [L^2]$ . This suggests that quantum action is fundamentally a measure of "spacetime area".

## C. Dimensional Equivalence and Its Implications for Fundamental Units

As a result of the dimensional collapse, within the context of the unified field, we have

$$[L] \equiv [T_{\text{ime}}] \equiv [M] \equiv [E] \equiv [Q] \equiv [T_{\text{emp}}], \quad (9)$$

meaning that, *within the theoretical model*, these quantities share a common dimensional basis. Strictly speaking, this does not invalidate the traditional distinction among meters, seconds, kilograms, joules, coulombs, and kelvins in everyday measurements or standard SI usage. Rather, it asserts that *when certain fundamental constants are equated and treated as direct conversion factors, one can recast these different units as numerically equivalent*:

$$1 \text{ m} \equiv 1 \text{ s} \equiv c^2 \text{ kg} \equiv 1 \text{ J} \equiv 1 \text{ C} \equiv k_B \cdot 1 \text{ K}. \quad (10)$$

This equivalence is neither arbitrary nor merely a notational trick; it reflects the idea that universal constants play the role of *natural conversion factors* between dimensions like length, time, mass, energy, charge, and temperature. In other words, once these constants are taken as fundamental geometric elements of the theory, the differences among standard SI units become a matter of labeling, rather than a manifestation of fundamentally different dimensions within the unified framework.

Hence, we embed the operational definitions of the meter, second, or kilogram into a broader dimensional structure, where what appear as separate units in standard SI can be seen, at a deeper theoretical level and normalized by the universal constants, as different expressions of the same underlying physical reality.

Under this viewpoint, universal constants lose their "arbitrary" character. They become the metric coefficients of a generalized physical geometry—effectively the "metric"—that converts one nominal unit into another. This unification thus provides a self-consistency check: if all these quantities truly emerge from the same resonant spacetime lattice, then setting them equal (normalized by the fundamental constants of nature) at the fundamental level should lead to a coherent, contradiction-free description of nature.

## Part II: Derivation of Fundamental Constants from Geometric-elastic Principles and Fundamental Laws

Building on the foundational principles and the resulting dimensional unification established in Subection IIC, we present a collection of derivations of fundamental physical constants and quantities obtained from first principles of the unified field and the quantum oscillator lattice framework, and through the direct application of well-established physical laws. These derivations serve a dual purpose: on one hand, they confirm the internal consistency of the framework; on the other, they provide new physical insights by revealing hidden connections among constants that traditionally appeared unrelated. We will show how constants of nature are not arbitrary scaling factors but geometric or elastic properties of the vacuum itself, not empirically isolated, but rather interconnected outputs of a deeper, modal-geometric structure of spacetime.

### IV. MODAL ACTIONS AND THE GEOMETRIC NATURE OF THE LAGRANGIAN

Having established the Unified Dimensional Framework, we now explore its consequences for the action,  $S$ , which governs all physical dynamics. We will show that the Lagrangian density,  $\mathcal{L}$ , is not an arbitrary function but has a necessary geometric form.

#### A. The Reference Action and its Fundamental Scale

We begin not by postulating the Lagrangian, but by deriving its required dimensions from the principles already established.

1. **The Dimension of Action:** In any physical system, the action,  $S$ , has dimensions of Energy  $\times$  Time. Within our framework, this becomes:

$$[S] = [E \cdot T] \equiv [L \cdot L] = [L^2] \quad (11)$$

This reveals that action is fundamentally a measure of **spacetime area**.

2. **The Dimension of the Lagrangian Density:** The action is defined as the integral of the Lagrangian density,  $\mathcal{L}$ , over a four-dimensional spacetime volume,  $d^4x$ .

$$S = \int \mathcal{L} d^4x \quad \implies \quad [S] = [\mathcal{L}] \cdot [d^4x] \quad (12)$$

Since  $[S] = [L^2]$  and  $[d^4x] = [L^4]$ , we can solve for the dimensions of  $\mathcal{L}$ :

$$[\mathcal{L}] = \frac{[S]}{[d^4x]} = \frac{[L^2]}{[L^4]} = [L^{-2}] \quad (13)$$

This is a profound result derived directly from our framework: *any fundamental Lagrangian density in this theory must have the dimensions of inverse area*. This result has immediate and deep implications, as the dimension of inverse area,  $[L^{-2}]$ , is precisely *the dimension of Gaussian or Ricci scalar curvature*. This finding directly connects our framework to the foundations of General Relativity, where the Ricci scalar,  $R$ , serves as the Lagrangian density in the Einstein-Hilbert action. Our derivation thus suggests that the choice of a curvature scalar as the Lagrangian is not arbitrary, but a necessary feature of a consistent unified theory. It implies that the Lagrangian density is not merely a function that describes dynamics on a spacetime manifold, but is, in a fundamental sense, a direct measure of the manifold's intrinsic curvature itself. Consequently, the Principle of Least Action is elevated to a Principle of Extremal Geometry, where the laws of physics emerge from spacetime's tendency to adopt a configuration that minimizes its own integrated curvature.

Now, we consider the simplest possible form for the baseline or "reference" Lagrangian density of the vacuum,  $\mathcal{L}_{\text{ref}}$ . In a theory built on fundamental unit scales, the most natural and non-trivial choice for a quantity with units of inverse area is the inverse of the unit area itself. We therefore define the reference Lagrangian as:

$$\mathcal{L}_{\text{ref}} := \frac{1}{(1 \text{ m})^2} \quad (14)$$

From this, we can calculate the *baseline modal action* for a unit 4-volume ( $d^4x = (1 \text{ m})^4$ ) in the reference regime:

$$\mathcal{S}_{\text{ref}} = \int \mathcal{L}_{\text{ref}} d^4x = \frac{1}{(1 \text{ m})^2} \cdot (1 \text{ m})^4 = 1 \text{ m}^2 \equiv 1 \text{ J} \cdot \text{s} \quad (15)$$

We have therefore derived a foundational equivalence of the theory:

$$1 \text{ m}^2 \equiv 1 \text{ J} \cdot \text{s} \quad (16)$$

This establishes a profound and direct identity between a purely geometric concept—spacetime area—and a purely dynamical one—action. This equivalence must be understood in a physical sense: the energy (E) required to create a deformation in the elastic vacuum, when sustained over a duration of time (T), sweeps out a total geometric

'deformation area' in the fabric of spacetime. In this unified view, action is no longer an abstract quantity that a system possesses, but rather it is the literal spacetime area associated with a physical process.

### B. The Electromagnetic Length Scale from Vacuum Properties

We now seek to determine the characteristic length scale,  $x_{EM}$ , for the electromagnetic (EM) mode. Rather than postulating a scaling rule, we will derive this length from the fundamental electromagnetic properties of the vacuum itself, as interpreted within our framework.

Our model is built on the structural isomorphism between mechanical and electromagnetic oscillators. In this analogy, mechanical displacement ( $x$ ) is the direct counterpart to electric charge ( $q$ ). Therefore, it is natural to identify the characteristic *length* of the EM mode,  $x_{EM}$ , with a fundamental *quantum of charge*, which we will call the "structural charge,"  $e_q$ .

$$x_{EM} \equiv e_q \quad (17)$$

We can construct this structural charge from the constitutive properties of the elastic vacuum  $\varepsilon_0$  and  $\mu_0$ . We define a "unitary vacuum capacitance" and a "unitary vacuum voltage" as follows:

- **Unitary Vacuum Capacitance ( $C_{vac}$ ):** The electric permittivity,  $\varepsilon_0$ , has SI units of Farads per meter. We define the capacitance of a unit segment of the vacuum as  $C_{vac} := \varepsilon_0 \cdot (1 \text{ m})$ .
- **Unitary Vacuum Voltage ( $V_{vac}$ ):** Similarly to the definition of the Unitary Vacuum Capacitance, we define the inductance of a unit segment of the vacuum as  $L_{vac} := \mu_0 \cdot (1 \text{ m})$ . As from the Inertial Equivalence Principle (4) and the Spacetime Equivalence Principle (8) we have  $[Inductance] \equiv [Mass] \equiv [Length]$ , the magnetic permeability  $\mu_0$  becomes dimensionally and conceptually equivalent to the unitary vacuum voltage,  $V_{vac} := \mu_0$ .

Another way to derive the same result uses Faraday's law, which describes the electromotive force induced by a time-varying magnetic flux:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L_{ind} \cdot \frac{dI}{dt}. \quad (18)$$

Using the base current  $I = 1 \text{ m/s}$  and the

base time  $t = 1 \text{ s}$ , and applying the equivalence  $L \equiv T$ , we find:

$$\mathcal{E} = \mu_0 \cdot 1 \text{ m} \cdot \frac{1 \text{ m/s}}{1 \text{ s}} = \mu_0 \quad (19)$$

and confirm that the magnetic permeability  $\mu_0$  characterizes the vacuum's minimal electromotive response to a unit deformation flow: the baseline voltage of the elastic field under quasi-static excitation.

Using the fundamental relation  $Q = C \cdot V$ , we can now construct the structural charge,  $e_q$ , from these vacuum properties:

$$e_q = C_{vac} \cdot V_{vac} = (\varepsilon_0 \cdot 1 \text{ m}) \cdot (\mu_0) = \frac{1 \text{ m}}{c^2} \quad (20)$$

By identifying the characteristic length with this structural charge,  $x_{EM} \equiv e_q$ , we have now derived the electromagnetic length scale from first principles:

$$x_{EM} = \frac{1 \text{ m}}{c^2} \equiv q_e \quad (21)$$

As a result, the scaling factor  $c^2$  is no longer an abstract "relativistic factor" but emerges directly from the product of the vacuum's fundamental elastic coefficients for electricity and magnetism.

*Remark: Spacetime Stiffness and the Analogy with  $E = mc^2$*

The result we have just derived,  $q_e \equiv x_{EM} = \frac{1 \text{ m}}{c^2}$ , reveals a profound symmetry when placed beside Einstein's iconic mass-energy relation,  $E = mc^2$ . Let us examine both in tandem:

$$\begin{aligned} x &= qc^2 \quad (\text{Geometric Deformation Equivalent}) \\ E &= mc^2 \quad (\text{Mass-Energy Equivalence}) \end{aligned} \quad (22)$$

Einstein's equation shows how mass is compressed energy—energy "crystallized" into curvature. Our relation shows how electric charge is a compressed deformation of spacetime—geometric strain encoded in the electromagnetic mode of the vacuum. In this view, charge is the residue of a dynamic process: the curling, twisting, or shearing of the elastic vacuum structure. Mass quantifies the inertial resistance to linear displacement (under gravity), and charge quantifies the resistance to rotational or vectorial distortion (under electromagnetism).

From this geometric-elastic perspective, the speed of light squared,  $c^2$ , emerges as a universal elastic modulus—a sort of "Young's modulus" of spacetime itself. Both equations then describe

a unified law of elastic response: the amount of underlying substance (energy or vacuum strain) required to produce a unit of geometric deformation (mass or charge). In this unified reading, charge and mass appear as dual modes of materialization from the same fabric: one compressive, the other torsional.

### C. The Action of the Electromagnetic Mode and the Origin of $\hbar$

Having derived the characteristic length scale of the electromagnetic mode, we can now determine its associated action. As the Lagrangian density must have dimensions of  $[L^{-2}]$  and the four-volume has dimensions of  $[L^4]$ , the scaling of  $x_{EM}$  dictates the scaling of  $\mathcal{L}_{EM}$  and  $d^4x_{EM}$ :

- The Lagrangian scales as  $\mathcal{L}_{EM} \propto 1/x_{EM}^2 \implies \mathcal{L}_{EM} = \mathcal{L}_{ref} \cdot c^4 = \frac{c^4}{(1\text{ m})^2}$ .
- The 4-volume scales as  $d^4x_{EM} \propto x_{EM}^4 \implies d^4x_{EM} = \frac{(1\text{ m})^4}{c^8}$ .

The total action for the electromagnetic mode is therefore:

$$\begin{aligned} \hbar &= \int \mathcal{L}_{EM} d^4x_{EM} = \frac{c^4}{(1\text{ m})^2} \cdot \frac{(1\text{ m})^4}{c^8} = \frac{1\text{ m}^2}{c^4} \\ &\equiv \frac{1\text{ J} \cdot \text{s}}{c^4} \approx 1.2 \times 10^{-34} \text{ J} \cdot \text{s} \end{aligned} \quad (23)$$

which closely approximates the measured value of the reduced Planck constant  $\hbar = 1.054 \times 10^{-34} \text{ J} \cdot \text{s}$ . The numerical discrepancy (about 15%) stems from complicated second-order corrections that will be addressed later on throughout the Paper, but at the end one has the more accurate equivalence

$$\hbar \equiv \left( \frac{1\text{ m}^2}{c^4} \cdot c \cdot \alpha^4 \right) \left( 1 + \frac{\alpha}{2\pi} + \dots \right)$$

### D. The Thermo-entropic Mode and the Cosmological Scale

Finally, we derive the properties of the third fundamental mode of the vacuum, which we associate with thermo-entropic and cosmological phenomena. We will derive its characteristic length scale,  $x_{th}$  from the fundamental quantum of energy within our framework.

1. **The Reference Quantum of Energy:** The energy of a fundamental quantum excitation is given by  $E = \hbar c/\lambda$ . We can define a "reference quantum of energy,"  $E_{q-ref}$ , by

evaluating this expression at the fundamental length scale of our system,  $\lambda = 1\text{ m}$ .

$$E_{q-ref} = \frac{\hbar c}{1\text{ m}} \quad (24)$$

2. **The Energy-Length Equivalence:** As established in our Unified Dimensional Framework, energy is dimensionally equivalent to length ( $[E] \equiv [L]$ ). Therefore, this reference quantum of energy is, fundamentally, a characteristic length.

3. **Derivation of the Thermo-entropic Length Scale:** We identify the characteristic length of this ultra-low-energy mode,  $x_{th}$ , with this fundamental quantum of energy:

$$x_{th} := E_{q-ref} = \frac{\hbar c}{1\text{ m}} \quad (25)$$

We can now calculate the value of this length scale by substituting our previously derived expression for the Planck constant,  $\hbar = 1\text{ m}^2/c^4$  (Eq. 23):

$$x_{th} = \frac{(1\text{ m}^2/c^4) \cdot c}{1\text{ m}} = \frac{1\text{ m}}{c^3} \quad (26)$$

As a result, we have that

$$x_{th} = \frac{1\text{ m}}{c^3} \implies \mathcal{L}_{th} = \frac{c^6}{1\text{ m}^2}, \quad d^4x_{th} = \frac{1\text{ m}^4}{c^{12}}$$

And the action for the thermo-entropic mode,  $S_{th}$ , is calculated as follows:

$$\begin{aligned} S_{th} &= \int \mathcal{L}_{th} d^4x_{th} = \frac{c^6}{(1\text{ m})^2} \cdot \frac{(1\text{ m})^4}{c^{12}} = \frac{1\text{ m}^2}{c^6} \\ &\equiv \frac{1\text{ J} \cdot \text{s}}{c^6} \approx 1.377 \times 10^{-51} \text{ J} \cdot \text{s} \end{aligned} \quad (27)$$

This quantity is numerically close to the observed value of the cosmological constant  $\Lambda \sim 10^{-52} \text{ m}^{-2}$ ; we will show later (XXIII B) how the cosmological constant itself may be interpreted as the action density of this *residual modal action* projected from ultra-low frequency deformations of the vacuum.

### Conclusion on Modal Actions

As a result, we have shown how some of the most important constants of nature arise not by empirical insertion, but as geometrically quantized projections of the same vacuum Lagrangian density over scaled modes of the same deformable elastic substrate. These results suggest that  $S_{ref}$ ,  $\hbar$ , and  $S_{th}$  act as modal Noether invariants—emergent from distinct projections of the same elastic vacuum structure—and that what we traditionally interpret as fundamental constants of nature are, in fact, geometric integrals over scaled deformations of a single quantized substrate.

## V. VACUUM ELECTROMOTIVE RESPONSES: $\mu_0$ , $K_e$ , AND $k_B$ AS MODAL RESONANCES FROM FARADAY DYNAMICS

Within the unified framework, we have seen how the current  $I$  represents a geometric deformation rate of the vacuum (IIB). Time and length are unified under the structural identification  $[L] = [T]$ , and current acquires the role of a velocity-like quantity:  $I \equiv \frac{L}{T}$ . In this section, we analyze how this dimensional symmetry allows us to reinterpret the vacuum permeability  $\mu_0$ , Coulomb's constant  $K_e$ , and Boltzmann's constant  $k_B$  as *effective electromotive responses*—emergent quantities characterizing the vacuum's elastic reaction to variations in structural currents over characteristic scales.

In order to proceed, we will use Faraday's law, which we have seen (18) that describes the electromotive force induced by a time-varying magnetic flux:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L_{\text{ind}} \cdot \frac{dI}{dt} \quad (28)$$

In the above expression, we will use the Unitary Vacuum inductance that we defined previously (IV B),  $L_{\text{vac}} := \mu_0 \cdot 1 \text{ m}$ , and we will perform different scalings on  $I$  and  $t$  to derive the effective electromotive responses of different regimes. Note that, for a natural unit length,

$$I \equiv \frac{L}{T} \equiv \frac{1}{T}$$

and thus, time is inversely proportional to current.

### A. Reference Regime — Gravitational mode.

Let us use the reference current  $I = 1 \text{ m/s}$  and the reference time  $t = 1 \text{ s}$ . Using the unitary vacuum inductance  $L_{\text{vac}} := \mu_0 \cdot 1 \text{ m}$ , and applying the equivalence  $L \equiv T$ , we find (as we already did in 19):

$$\mathcal{E} = \mu_0 \cdot 1 \text{ m} \cdot \frac{1 \text{ m/s}}{1 \text{ s}} = \mu_0 \quad (29)$$

Thus, the magnetic permeability  $\mu_0$  characterizes the vacuum's minimal electromotive response to a unit deformation flow: the baseline voltage of the elastic field under quasi-static excitation. Note that is the same derivation we have already performed in (IV B).

### B. Electromagnetic mode (scaled by $c$ ).

Scaling the current as  $I = c$  implies  $t = \frac{1}{c} \text{ s}$ , and using the same  $L_{\text{vac}}$ :

$$\mathcal{E} = \mu_0 \cdot 1 \text{ m} \cdot \frac{c}{\frac{1}{c}} = \mu_0 c^2 = 4\pi K_e \quad (30)$$

We recover Coulomb's constant  $K_e$  as a relativistically scaled version of  $\mu_0$ , confirming its nature as a voltage response under rapid propagation of deformation.

### C. Thermo-entropic mode (scaled by $1/c$ )

Scaling the current as  $I = 1/c$  yields  $t = 1 \text{ s} \cdot c$ . Then:

$$\mathcal{E} = \mu_0 \cdot 1 \text{ m} \cdot \frac{\frac{1}{c}}{1 \text{ s} \cdot c} = \frac{\mu_0}{c^2} \approx 1.4 \times 10^{-23} \quad (31)$$

which coincides numerically with Boltzmann's constant  $k_B$ , up to second-order terms. This identifies  $k_B$  as the thermo-entropic counterpart to  $\mu_0$  and  $K_e$ : an emergent voltage under slow deformation flux, consistent with the azimuthal thermal mode  $\vec{T}$  derived later from the unified tensor field (122).

These results confirm that  $\mu_0$ ,  $K_e$ , and  $k_B$  are not independent constants, but rather *modal projections* of the same elastic tensor structure. Each emerges from Faraday-like dynamics under a distinct current/time scaling regime:

- **Gravitational:** base mode, with  $\mathcal{E} = \mu_0$ ,
- **Electromagnetic:** relativistic excitation, with  $\mathcal{E} = \mu_0 c^2 = 4\pi K_e$ ,
- **Entropic:** dissipative/thermal excitation, with  $\mathcal{E} = \mu_0/c^2 \equiv k_B$ .

In this view, all three constants describe the same underlying stiffness of the vacuum, observed at different frequencies of excitation. This directly supports the modal structure proposed in XVIII B, where each field expression  $\vec{\Phi}_X(r) = \frac{\mu_0}{4\pi r} \cdot C_X \cdot \hat{e}_X$  arises from a specific symmetry and energy scale of the elastic vacuum (XXII B).

As a result, Faraday's law, when interpreted in modal terms, provides a unified operational principle: *electromotive response emerges from geometric resistance to deformation*. The constants  $\mu_0, K_e, k_B$  are expressions of the same elastic modulus  $\mu_0$ , rescaled by the velocity  $I$  of excitation. This unifies electromagnetic, gravitational, and thermo-entropic interactions as frequency-dependent modes of a single deformable spacetime substrate.

## VI. VACUUM IMPEDANCE $Z_0$ AND AMPÈRES LAW

In the quasi-static regime, Ampère's law (ignoring displacement currents) relates the magnetic field to the current  $I$  via

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I = \mu_0 c = Z_0 \quad (32)$$

This identity reveals a fundamental insight within our framework: the vacuum impedance  $Z_0 = \mu_0 c$  is not merely an electromagnetic constant, but a quantized circulation of the magnetic field in response to the current  $I = c$ . Under the quasi-static regime, Ampère's law reduces to a direct proportionality between the magnetic field circulation and this structural current, leading to the elegant equivalence  $\oint \vec{B} \cdot d\boldsymbol{\ell} = Z_0$ .

In this light,  $Z_0$  characterizes the vacuum's intrinsic resistance to topological twisting — a kind of "circulatory stiffness" — analogous to how  $\mu_0$  represents linear stiffness to deformation. When interpreted through the lens of dimensional collapse, where both  $\mu_0$  and  $I$  are dimensionless, this equation becomes a quantization condition: the magnetic excitation mode  $\vec{B}$  must integrate to a unit value. This supports the interpretation of  $\vec{B}$  as a topological mode of the unified field, constrained by geometric and symmetry conditions of the vacuum itself. In this sense, magnetism becomes a circulatory polarization of spacetime, fixed by the internal geometry and encoded in the impedance of the vacuum medium.

This reinterpretation of  $Z_0 = \mu_0 c$  as a quantized circulation constant aligns with well-established principles of flux quantization in field theory. For instance, in quantum systems such as superconductors, magnetic flux is quantized in units of  $\Phi_0 = h/2e$ , while in classical electromagnetism, integral forms of Maxwell's equations impose global constraints on field flux. The unified modal formulation (XXII B) extends these principles by embedding them into the geometry of spacetime: the field circulation  $\oint \vec{B} \cdot d\boldsymbol{\ell}$  must yield a fixed value determined by the internal structure of the vacuum. Hence, the elastic tensor framework not only recovers the standard laws, but elevates them to quantization conditions on the permissible excitations of the vacuum's geometry. This reinforces the central insight of (XVIII B): that all physical interactions arise as topologically constrained modal deformations of a single Lorentz-invariant field tensor  $\mathcal{G}_{\mu\nu}$ .

## VII. ENERGY CONSISTENCY CHECKS: SOME FUNDAMENTAL DERIVATIONS

### A. The LC oscillator-like equipartition and a basic field-theoretic setting

Consider the energy stored in a capacitor and inductor:

$$E_{LC} = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 \quad (33)$$

and substitute:

- $C = \varepsilon_0 \cdot 1 \text{ m}$  is the capacitance, consistent with the definition and SI units of  $\varepsilon_0$ .
- We can apply Ohm's Law to derive  $V = I \cdot R = c \cdot Z_0 = \frac{1}{\varepsilon_0}$  as some voltage,
- $I = c$  is the current within the electromagnetic mode in the context of the unified field.

Substituting these values yields

$$E_{LC} = \frac{1}{2}(\varepsilon_0 \cdot 1 \text{ m}) \cdot \frac{1}{\varepsilon_0^2} + \frac{1}{2}L \cdot c^2 \quad (34)$$

This discrete LC system serves as a localized analogy for the vacuum's elastic behavior. To capture this oscillatory structure in a continuous, field-theoretic setting, let  $\Phi(x)$  denote a *field* -in general, it can be a multi-component scalar, vector, or tensor, but for illustrative purposes, we treat it here as a single real scalar field-. Include just the two couplings (or 'rigidities') usual for harmonic oscillatory systems. Then, one has the following *minimal* Lagrangian density:

$$\mathcal{L}(\Phi) = \frac{1}{2} \kappa_1 (\partial_t \Phi)^2 - \frac{1}{2} \kappa_2 (\nabla \Phi)^2 \quad (35)$$

Here,

- $\kappa_1$  controls the 'inertial' or kinetic response of the field mode,
- $\kappa_2$  represents the elastic/spatial rigidity of the field mode.

Note that the requirement

$$\kappa_1 = \kappa_2$$

is the very condition that makes the field equation into the Lorentz-invariant wave equation

$$\square \Phi = 0,$$

as for any plane-wave solution

$$\Phi(t, \mathbf{x}) \propto e^{i(\omega t - \mathbf{k} \cdot \mathbf{x})}$$

this equality forces  $\omega^2 = |\mathbf{k}|^2$ , and one finds at every point (or on average over a cycle)

$$\frac{1}{2} \kappa_1 (\partial_t \Phi)^2 = \frac{1}{2} \kappa_2 (\nabla \Phi)^2$$

In other words, Lorentz symmetry *kinematically* guarantees that each mode carries exactly half its energy in “kinetic” form and half in “gradient” form. We may therefore adopt the usual LC-circuit equipartition

$$C V^2 = L I^2$$

We can solve (34) for the inductance  $L$  to get:

$$L = \frac{C V^2}{I^2} = \frac{1 m}{\varepsilon_0 c^2} = \mu_0 \cdot 1 m \quad (36)$$

Which serves as a consistency check, as our proposed quantum of inductance is indeed  $\mu_0 \cdot 1 m$  (IV B). And, substituting with  $L$  and calculating, we get that

$$E_{LC} = \frac{1 m}{\varepsilon_0} = \mu_0 \cdot 1 m \cdot c^2 = 4\pi K_e \cdot 1 m$$

The result  $E_{LC} = 4\pi K_e \cdot 1 m$ , as within the unified field we have that  $\left[\frac{1}{\varepsilon_0}\right] = [4\pi K_e] \equiv [V]$  and  $1 m \equiv 1 C$ , can be interpreted as the quantum of electrostatic energy.

This expression not only confirms the internal consistency of our dimensionally collapsed framework, but also shows that *the requirement  $\kappa_1 = \kappa_2$ —which ensures Lorentz invariance in the scalar field model—corresponds physically to energetic equipartition between electric and magnetic modes (or kinetic and potential modes in the oscillator analogy).*

In this view, the field equation  $\square\Phi = 0$  represents the fundamental wave dynamics of the unified vacuum, and its LC analogue illustrates how energy is stored and propagated through field excitations. We will postulate in (XVIII B) that each field mode that we propose (e.g. electromagnetic, thermo-entropic) can be modeled as a scalar excitation with effective rigidity  $\kappa$ , matched to the appropriate constant ( $\varepsilon_0$ ,  $\mu_0$ ,  $G$ ,  $k_B$ ) depending on the physical context.

Thus, the scalar field model drafted above does more than reproduce known equations—it captures the essential modal structure of the symmetric fundamental tensor  $\mathcal{G}_{\mu\nu}$  (XVIII B), and justifies, from energetic and variational grounds, the emergence of field equations and constants from a deeper elastic structure. In this light, the equality  $\kappa_1 = \kappa_2$  becomes not a mere condition for symmetry, but a *physical principle of balance*: the propagation of deformation through the vacuum respects a universal ratio between inertial response and spatial rigidity—a signature of a fundamentally oscillatory spacetime.

## B. The vacuum energy density from the RLC circuit analogy and peak energy

In (33), we can naturally set  $L = \mu_0 \cdot 1 m$  and  $C = \varepsilon_0 \cdot 1 m$  as the inductance and capacitance of the system (IV B, IV B). Also, we can set the scaled angular frequency for the electromagnetic regime (V B)

$$\omega_0 = \frac{c}{1 s}$$

For sinusoidal oscillations, we can express the current  $I$  as:

$$I(t) = -Q_0 \cdot \omega_0 \sin(\omega t)$$

where  $Q_0$  is the maximum charge on the capacitor. Thus, we can see that the peak current  $I_{\max}$  (the maximum value of  $I(t)$ ) is:

$$I_{\max} = Q_0 \cdot \omega_0$$

Then, with the equivalence  $e = Q_0$  (where  $e$  is the elementary electric charge) and  $\frac{c}{1 s} = \omega_0$ , we have that the maximum current of the system is given by

$$I_{\max} = \frac{e \cdot c}{1 s} \quad (37)$$

The total energy of a fundamental mode of the quantum vacuum does not behave like a classical oscillator. Its total energy is not a time-averaged quantity, but is instead determined by the sum of the peak potential energies that its modal components can harbor. Thus, we define the peak potential energy of the electric mode as:

$$U_{E,\text{peak}} = \frac{e^2}{C} = \frac{e^2}{\varepsilon_0 \cdot 1 m}$$

And the peak potential energy of the magnetic mode as:

$$U_{M,\text{peak}} = L I^2 = (\mu_0 \cdot 1 m) \left(\frac{e \cdot c}{1 s}\right)^2 = \frac{\mu_0 e^2 c^2}{1 m}$$

The total energy of the fundamental vacuum mode is the sum of these maximum potentials:

$$E_{\text{total}} = U_{E,\text{peak}} + U_{M,\text{peak}} = \frac{e^2}{\varepsilon_0 \cdot 1 m} + \frac{\mu_0 e^2 c^2}{1 m} \quad (38)$$

This definition reflects the nature of the vacuum as a quantum field where the electric and magnetic modes are intrinsically coupled, and their energetic potential coexists persistently, unlike the energy transfer in a classical oscillator.

Importantly, using the equivalence  $1 m \equiv 1 s \equiv 1 C$ , one gets that  $[U_{E,\text{peak}}] \equiv [U_{M,\text{peak}}] \equiv [Kg]$ . And, substituting with the CODATA values of the

fundamental constants, one gets an approximate value of  $E_{\text{total}} \approx 5.75 \times 10^{-27} \text{ kg}$ . This value aligns with the measurements of the vacuum energy density  $\rho_{\text{vac}}$  obtained by the Planck Collaboration in 2015 [18]. We can interpret this value as the fundamental mass-energy contained within the characteristic unit volume of the vacuum,  $V_{\text{ref}} = (1 \text{ m})^3$ , so  $\rho_{\text{vac}} \equiv \left[ \frac{E_{\text{total}}}{V_{\text{ref}}} \right]$ .

The above underscores again both the internal consistency and the potential theoretical power of the established framework.

### C. Derivation of Rayleigh dissipation from core identifications and Hooke's law

Consider some total dissipated electromagnetic energy due to resistance. Using the Rayleigh dissipation function

$$F = \frac{1}{2}RI^2, \quad (39)$$

the instantaneous power dissipation in the system is given by:

$$P_{\text{dis}}(t) = \frac{dE_{\text{dis}}}{dt} = RI^2 \quad (40)$$

Substituting the defined core parameters for the electromagnetic regime ( $R = Z_0$  and  $I = c$ ), we have

$$P_{\text{dis}}(t) = Z_0 \cdot c^2 \quad (41)$$

To determine the total energy dissipated over the characteristic time interval  $\tau$ , we integrate this power dissipation from 0 to  $\tau$ :

$$E_{\text{dis}} = \int_0^\tau P_{\text{dis}}(t) dt \quad (42)$$

Using the characteristic time in the electromagnetic regime  $\tau = \frac{1s}{c}$ , we evaluate the integral:

$$E_{\text{dis}} = Z_0 \cdot c^2 \cdot \frac{1 \text{ s}}{c} = \frac{1}{\varepsilon_0} \cdot 1 \text{ s} \quad (43)$$

The above is dimensionally consistent within the framework of the unified field, as  $\left[ \frac{1}{\varepsilon_0} \right] \equiv [V]$  and  $1 \text{ s} \equiv 1C$ .

To confirm the potential power of the geometric-elastic framework that we are developing, the above expression admits a direct mechanical analogy with Hooke's law in its linear form, whose absolute value expression for total energy yields  $|E| = kx^2$ . Applying the correspondence  $k \equiv \frac{1}{C}$  between mechanical and electrical oscillatory systems, we can identify the inverse vacuum

permittivity per meter,  $\frac{1}{\varepsilon_0 \cdot 1 \text{ m}}$ , with an effective *stiffness* constant  $k$  of the vacuum —representing its resistance to deformation or response under excitation— while we also have established that  $1 \text{ s} \equiv 1 \text{ m}$ , which naturally corresponds to a spatial displacement  $x$ . The resulting structure,  $E_{\text{dis}} = kx^2 = \frac{1}{\varepsilon_0} \cdot 1 \text{ s}$ , underscores how the unified field behaves as an elastic medium, in which energy is linearly related to displacement via an intrinsic stiffness.

## VIII. THE FINE-STRUCTURE CONSTANT $\alpha$ AS A DAMPING RATIO $\zeta$ AND A LORENTZ-LIKE FACTOR

The fine-structure constant  $\alpha$  [19] can be defined as the ratio of two energies:

- the energy needed to overcome the electrostatic repulsion between two electrons a distance of  $d$  apart
- the energy of a single photon of wavelength  $\lambda = 2\pi d$  (or of angular wavelength  $d$ )

Therefore, we have that

$$\begin{aligned} \alpha &= \left( \frac{e^2}{4\pi\varepsilon_0 d} \right) / \left( \frac{hc}{\lambda} \right) \\ &= \frac{e^2}{4\pi\varepsilon_0 d} \times \frac{2\pi d}{hc} = \frac{e^2}{4\pi\varepsilon_0 d} \times \frac{d}{\hbar c} \\ &= \frac{e^2}{4\pi\varepsilon_0 \hbar c} \end{aligned} \quad (44)$$

Other hand, in the context of an RLC circuit, the quality factor or Q factor [20] is a dimensionless parameter that describes how underdamped an oscillator or resonator is. It is defined as the ratio of the initial energy stored in the resonator to the energy lost in one radian of the cycle of oscillation. Therefore, we have that

$$\begin{aligned} Q &\stackrel{\text{def}}{=} 2\pi \times \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} \\ &= 2\pi f_r \times \frac{\text{Energy stored}}{\text{Power loss}} \\ &= \omega_0 \times \frac{\text{Energy stored}}{\text{Power loss}} \end{aligned} \quad (45)$$

Where  $f_r$  is the resonance frequency.

The fine-structure constant can be written as

$$\alpha = \frac{1}{2}Z_0\sigma,$$

where  $Z_0 = \mu_0 c = \frac{1}{\varepsilon_0 c}$  is the vacuum impedance and  $\sigma = \frac{e^2}{\hbar}$  is the conductance quantum [21]. It follows that

$$Z_0\sigma = 2\alpha,$$

which can be interpreted as the intrinsic energy loss characteristic per radian for the vacuum medium itself. Thus, one can naturally define a vacuum quality factor as

$$Q = \frac{1}{Z_0 \sigma} = 2\pi \times \frac{\varepsilon_0 \hbar c}{e^2} = \frac{1}{2\alpha}.$$

where we can identify  $E_{\text{stored}} = \frac{\hbar c}{\lambda}$  and  $E_{\text{dissipated}} = \frac{e^2}{\varepsilon_0 \lambda}$ .

The appearance of these two energies follows directly from well-established properties of a single electromagnetic vacuum mode of wavelength  $\lambda$  and the framework we have discussed throughout this Paper. On the one hand, the most fundamental quantum excitation of an electromagnetic mode is a single photon. The energy of this photon, which represents the total energy “stored” in the oscillating mode for this process, is given by the Planck-Einstein relation:

$$E_{\text{stored}} = \hbar \omega = \frac{\hbar c}{\lambda}$$

where the energy of a single photon of wavelength  $\lambda = 2\pi d$  is used. This interpretation defines  $E_{\text{stored}}$  as the energy of the fundamental quantum of light that populates and defines the oscillation within that vacuum mode.

On the other hand, within our elastic vacuum formalism, it is natural to propose a dissipated energy analogous to Hooke’s law. As we will demonstrate below with a rigorous calculation based on Larmor’s formula, this simple analogy predicts with great accuracy the energy dissipated by a quantum dipole.

- Using Hooke’s Law, we can identify the dissipated energy using the formula  $|E| = |kx^2|$  where  $k$  is the elasticity constant, and  $x$  the displacement [22]. As in the context of the unified field we have identified  $k = \frac{1}{C} = \frac{1}{\varepsilon_0 \lambda}$ , and the displacement  $x$  with the elementary charge  $e$ , we get that  $E_{\text{dissipated}} = \frac{e^2}{\varepsilon_0 \lambda}$ .
- Alternatively, consider an oscillating dipole  $p(t) = p_0 \cdot \cos(\omega t)$  with amplitude  $p_0 = e \cdot d$ , where  $d = \lambda/(2\pi)$  is the separation and  $\omega = c/d$  is the angular frequency. Using Larmor formula for time-averaged power radiated by an oscillating dipole [23]:

$$\langle P \rangle = \frac{p_0^2 \omega^4}{12\pi \varepsilon_0 c^3} \quad (46)$$

we can derive the energy dissipated per cycle (period  $T = 2\pi/\omega$ ):

$$E_{\text{dissipated}} = \langle P \rangle \times T = \left( \frac{p_0^2 \omega^4}{12\pi \varepsilon_0 c^3} \right) \left( \frac{2\pi}{\omega} \right) \quad (47)$$

And one finally gets that

$$E_{\text{dissipated}} = \frac{\pi e^2}{3\varepsilon_0 \lambda} \approx 1.05 \cdot \frac{e^2}{\varepsilon_0 \lambda} \quad (48)$$

Thus, both energies can be derived from first-principles and lead directly to

$$Q = 2\pi \times \frac{E_{\text{stored}}}{E_{\text{dissipated}}} = 2\pi \times \frac{\hbar c/(\lambda)}{e^2/(\varepsilon_0 \lambda)} = \frac{1}{2\alpha},$$

For an underdamped oscillator, the damping ratio is defined as

$$\zeta = \frac{1}{2Q},$$

which leads directly to

$$\boxed{\zeta = \alpha}$$

#### A. Interpreting $c$ in terms of the Damped Resonant Frequency of the System

In a standard underdamped oscillator model [24–26], the damped frequency  $\omega_d$  is given by

$$\omega_d = \omega_0 \sqrt{1 - \zeta^2}, \quad (49)$$

where  $\omega_0$  is the *undamped* resonant (natural) frequency of the system, and  $\zeta$  is the damping ratio.

We can associate these frequencies with propagation speeds by multiplying each angular frequency by the reference length within our framework, of *one meter*, yielding speeds in units of  $\text{m s}^{-1}$ . Denoting:

$$v_{\text{damped}} = \omega_d \times 1 \text{ m}, \quad v_{\text{undamped}} = \omega_0 \times 1 \text{ m},$$

we can identify  $v_{\text{damped}}$  with the *measured* speed of light, conventionally denoted by  $c$ . In other words,

$$c = v_{\text{damped}} = \omega_d \times 1 \text{ m}.$$

From Eq. (49), we thus have

$$c = \omega_0 \cdot 1 \text{ m} \sqrt{1 - \zeta^2}$$

Or, equivalently,

$$c_{\text{measured}}^2 = c_{\text{real}}^2 (1 - \alpha^2) \quad (50)$$

Which, solving for  $\zeta$ , can be rewritten as

$$\zeta = \alpha = \sqrt{1 - \frac{c_{\text{measured}}^2}{c_{\text{real}}^2}} \quad (51)$$

Note the similarity of the above expression with the reciprocal of the Lorentz factor formula [27].

Thus, the fine-structure constant  $\alpha$  can be regarded as the reciprocal of a “Lorentz-like” factor via

$$\zeta = \alpha = \frac{1}{\gamma} = \sqrt{1 - \frac{c_{\text{measured}}^2}{c_{\text{real}}^2}}. \quad (52)$$

These two views—the *damped oscillator* analogy for electromagnetic propagation and the *Lorentz-like* factor interpretation for  $\alpha$ —are not only compatible, but in fact reinforce each other:  $\alpha$  emerges as a geometric or relativistic “scaling factor” that governs attenuation in the oscillatory unified field, connecting electromagnetic propagation and the quantum vacuum’s dissipative properties.

**B. Dissipative duality: roles of the damping factor  $\zeta = \alpha$  and the reciprocal of the quality factor  $\frac{1}{Q} = 2\alpha$**

*Physical Origin: The Fermionic g-factor*

The theoretical framework that we will develop reveals an apparent duality in the vacuum’s dissipative response. On one hand, the damping that affects wave propagation manifests as a relativistic damping ratio  $\zeta = \alpha$ . On the other hand, the attenuation of energy in quantum interactions, quantified by the quality factor  $Q$ , is governed by  $1/Q = 2\alpha$ .

We propose that this duality is not a contradiction but a reflection of a deeper physical principle: *the factor of 2 originates from the Landé g-factor ( $g_e$ ) of a fundamental, spin-1/2 fermionic excitation of the vacuum*, whose value predicted by the Dirac Equation is precisely  $g_e = 2$ . This hypothesis establishes a fundamental distinction between two dissipative regimes:

- **Bosonic Propagation Damping ( $\zeta$ ):** The factor  $\zeta = \alpha$  represents the intrinsic damping experienced by a bosonic excitation (such as a spin-1 photon) as it propagates through the elastic vacuum medium. It is a pure measure of spacetime’s “viscosity.”
- **Fermionic Interaction Damping ( $1/Q$ ):** The factor  $1/Q = g_e\alpha \approx 2\alpha$  governs processes that involve the fermionic nature of the vacuum’s excitations. This includes the “dressing” of a bare charge to form a stable particle (like the electron) or the energy transfer that gives rise to a quantum excitation. In these cases, the dissipation depends not only on the base viscosity ( $\alpha$ ) but is also amplified by the intrinsic spin-1/2 response ( $g_e = 2$ ) of the excitation.

Far from being an electron-centric hypothesis, this interpretation posits that  $g_e = 2$  is a topological property of the fermionic modes of the vacuum lattice itself. Particles like the electron simply inherit this fundamental characteristic. This idea is consistent with the treatment of the anomalous magnetic moment ( $a_e = (g_e - 2)/2$ ) as a higher-order correction to the same dissipative mechanism.

*Analogy with the RLC Circuit: Transient Damping vs. Resonant Dissipation*

This physical distinction between  $\zeta$  and  $Q$  finds a powerful analogue in the behavior of a classical RLC circuit, which strengthens its conceptual justification:

- The **damping ratio** ( $\zeta$ ) in an RLC circuit determines the system’s *transient response* (i.e., how a free oscillation decays). Analogously,  $\zeta = \alpha$  in our model governs the decay of a wave propagating freely through the vacuum.
- The **quality factor** ( $Q$ ), in contrast, describes the *steady-state or resonant response*. It is defined as the ratio of reactive impedance to dissipative resistance, quantifying the energy loss *per cycle* in an established oscillation. Analogously,  $1/Q = g_e\alpha$  in our model governs the energy dissipation in a stable interaction, such as that which constitutes a “dressed” particle, where vacuum energy is continuously stored and dissipated.

Therefore, the vacuum is interpreted as a medium possessing a base friction for wave propagation ( $\zeta = \alpha$ ) and a distinct, amplified dissipative resistance for fermionic interactions ( $1/Q = g_e \cdot \alpha$ ). This interpretation resolves the apparent duality by unifying the vacuum’s relativistic properties with its quantum, spin-dependent interactions.

**C. Interpretation and discussion on Vacuum Damping**

Throughout this Paper, we are proposing that the quantum vacuum itself acts as a structured, elastic-dissipative medium. In this picture, the vacuum consists of fluctuating virtual excitations with internal degrees of freedom that collectively endow it with both stiffness (reactive elasticity) and finite relaxation time (viscosity). A propagating electromagnetic wave then loses energy—not into real particles, but into the hidden structure of the vacuum—via coupling to these degrees of freedom. This loss manifests macroscopically

as a damping ratio  $\zeta$ , which we identify with the dimensionless fine-structure constant via  $\zeta = \alpha$ . Such a damping constant is natural if one treats the vacuum as an ensemble of coupled oscillators or as an emergent condensed-matter system, as suggested by various approaches to quantum gravity and emergent spacetime [28–30]. The resulting reduction in propagation speed is then not a kinematic effect, but a first-principles consequence of quantum back-reaction. This allows us to interpret the measured speed of light  $c$  as a damped, effective velocity arising from the underlying dissipative structure of the vacuum.

Importantly, this view does not violate local Lorentz invariance. All local observers measure the same effective speed  $c = c_{\text{measured}}$ , and all physical laws remain Lorentz-invariant in that frame. The distinction between  $c_{\text{real}}$  and  $c$  thus becomes a global, geometric feature of the vacuum — akin to how curvature encodes gravitational effects in general relativity. In this case, however, the “curvature” is not geometric but modal: a manifestation of the vacuum’s internal damping modes, whose excitation state defines a preferred frame only at a topological level, not at the level of measurable kinematics. Analogous to an RLC circuit, where the “natural” frequency  $\omega_0$  is never directly observed but rather only inferred through modeling, the proposed  $c_{\text{real}} > c$  does not admit superluminal information transfer, and thus poses no contradiction to special relativity or experiment.

#### D. Final notes

Higher-order radiative effects — e.g. the electron’s anomalous magnetic moment  $a_e = \alpha/2\pi$  — can be viewed as additional layers of the same dissipative mechanism. The damping encoded in the fine-structure constant  $\alpha$  would be the first-order manifestation of how the vacuum’s oscillator lattice “bleeds” energy back into itself through quantum fluctuations, and the anomalous magnetic moment can be viewed, in our framework, as the simplest radiative attenuation of a bare “undamped” coupling by the lattice’s elastic resistance. Higher-order Feynman diagrams then correspond to more intricate couplings among modes of the vacuum, each contributing successive powers of  $\alpha$ .

More generally, it implies that any relation among fundamental constants—whether in electromagnetism, gravitation or thermodynamics—must be dressed by a universal, dimensionless form factor  $\Xi_{\text{eff}}(\alpha)$  that encodes the accumulated

effect of loop-induced damping within the vacuum lattice. In this way, radiative corrections are not mere perturbative afterthoughts, but the fingerprint of the same elastic and dissipative structure that unifies all fields at their quantum origin.

In this view, *the fine-structure constant  $\alpha$  becomes not merely a coupling constant, but a unifying signature of modal attenuation across all field interactions* — electromagnetic, gravitational, and thermo-entropic alike.

### IX. SYNTHESIS OF CONSTANTS AND THE VACUUM’S FUNDAMENTAL EQUATION

In this section, we demonstrate how the framework of the elastic vacuum leads to a profound synthesis between the thermal, electromagnetic, and quantum domains. We begin by presenting a remarkable numerical relationship as a conjecture, which we then derive from the first principles of our theory.

#### A. A Numerical Conjecture and the Elementary Charge

Let us examine the following combination of fundamental constants:

$$\frac{k_B \cdot 2\alpha}{\mu_0}$$

Using the accepted CODATA values for Boltzmann’s constant ( $k_B$ ), the fine-structure constant ( $\alpha$ ), and the vacuum permeability ( $\mu_0$ ), this expression yields:

$$\begin{aligned} & \frac{(1.3806 \times 10^{-23}) \cdot 2 \cdot (0.007297)}{4\pi \times 10^{-7}} \\ & \approx 1.603 \times 10^{-19} \end{aligned} \quad (53)$$

This value is very close to the measured value of the *elementary charge*,  $e$ . This striking numerical alignment strongly suggests the existence of an underlying physical relationship:

$$e \equiv \frac{k_B \cdot 2\alpha}{\mu_0}. \quad (54)$$

In the following subsection, we will show that this is not a coincidence, but a direct consequence of our model.

#### B. Derivation of the Physical Charge from a Damping Principle

Our theory posits that measured physical quantities emerge from “bare” quantities intrinsic to

the vacuum, which are then "dressed" by the vacuum's universal damping response, quantified by  $\frac{1}{Q} = 2\alpha$  as the first-order term.

1. **The Bare Charge ( $e_q$ ):** As established in Section IV B, the structural or "bare" charge of the vacuum,  $e_q$ , is constructed from its constitutive properties:  $e_q = \varepsilon_0 \mu_0 \cdot 1 \text{ m} = \frac{1 \text{ m}}{c^2}$ .
2. **The Thermo-Entropic Voltage ( $k_B$ ):** As derived in Section V C, the Boltzmann constant is interpreted as the effective voltage of the vacuum's lowest-energy (thermo-entropic) mode:  $k_B \equiv \frac{\mu_0}{c^2}$ .
3. **The Bare Relationship:** By combining these two principles, we can formulate a "bare" relationship. From the expression for  $k_B$ , we can state  $1/c^2 \equiv k_B/\mu_0$ . Substituting this into the equation for the bare charge gives:

$$e_q = \frac{1 \text{ m}}{c^2} \equiv \frac{k_B \cdot 1 \text{ m}}{\mu_0}.$$

This equation relates the bare charge to the thermal energy scale, noting that  $1 \text{ m} \equiv 1 \text{ K}$  within our framework.

4. **The Physical Charge ( $e$ ):** The observed physical charge,  $e$ , is the bare charge  $e_q$  after being "dressed" by the vacuum's dissipative response. The factor governing this process is  $2\alpha$  in a first-order approximation. We therefore propose the relationship:

$$e \equiv e_q \cdot (2\alpha) \equiv \left( \frac{k_B \cdot 1 \text{ m}}{\mu_0} \right) \cdot 2\alpha. \quad (55)$$

Where we exactly recover the numerical relationship conjectured in Eq. (54).

### C. From the equipartition theorem to the harmonic oscillator energy and the fundamental equation

One can observe another remarkable numerical concordance. The term  $\mu_0 \cdot e$  is numerically  $\approx 2.01 \times 10^{-25} \text{ J} \cdot \text{s/m}$ , while the quantum reference energy per unit length,  $hc/(1 \text{ m})$ , is  $\approx 1.98 \times 10^{-25} \text{ J} \cdot \text{s/m}$ . This suggests a single, powerful **Fundamental Equation of the Vacuum**:

$$\boxed{\mu_0 \cdot e \equiv k_B \cdot 2\alpha \cdot 1 \text{ K} \equiv \frac{hc}{1 \text{ m}}} \quad (56)$$

where the second-order terms have been omitted, and a more accurate approximation would be given

by

$$\begin{aligned} & \mu_0 \cdot e_q (2\alpha + \frac{\alpha}{2\pi} + \dots) \\ & \equiv k_B \cdot 1 \text{ K} (2\alpha + \dots) \\ & \equiv \frac{hc}{1 \text{ m}} (1 + 2\alpha + \dots) \end{aligned} \quad (57)$$

This relations provide a profound synthesis between thermal, quantum, electromagnetic, and geometric domains. The fundamental equation encapsulates, in a single concise expression, this Paper's overarching claim that multiple "fundamental constants" can be viewed as interrelated manifestations of an underlying quantum-oscillatory vacuum.

### D. A "Damped Equipartition" Principle

The final step is to provide a physical origin for the relationships synthesized in the Fundamental Equation. We can achieve this by re-examining the connection between classical thermal energy and quantum excitation energy through the lens of our dissipative vacuum model.

The standard equipartition theorem states that the average thermal energy available from a classical harmonic oscillator is  $\langle E \rangle = k_B T$  [31]. In an ideal, non-dissipative universe, one might postulate that this entire energy is available to create a quantum excitation of energy  $E = \hbar\omega$ .

However, in our framework, the vacuum is not an ideal medium; it is inherently dissipative. Any energy transfer is "damped" by the vacuum's structure, a process quantified at first-order by the factor  $2\alpha$ . Therefore, we propose a *Damped Equipartition Principle*: the *effective* thermal energy that is physically available for a quantum excitation is the classical thermal energy attenuated by the vacuum's damping factor:

$$E_{\text{dressed}} = (k_B T) \cdot (2\alpha)$$

This "dressed" energy is what a quantum system can actually draw from the thermal bath. For a fundamental excitation of the vacuum lattice, this energy must match the quantum of energy required,  $E_{\text{quantum}} = \hbar\omega = \frac{hc}{\lambda}$ . By equating the dressed thermal energy with the quantum excitation energy, we get:

$$k_B T \cdot 2\alpha = \frac{hc}{\lambda}$$

This equation elegantly connects thermodynamics ( $k_B T$ ), quantum mechanics ( $hc/\lambda$ ), and the vacuum's dissipative structure ( $2\alpha$ ). To test it

against our *Fundamental Equation*, we evaluate it at the reference scale of our framework, setting the characteristic length for both temperature and wavelength to one meter ( $T \equiv 1\text{ K} \rightarrow 1\text{ m}$ , and  $\lambda = 1\text{ m}$ ):

$$k_B \cdot (1\text{ K}) \cdot 2\alpha = \frac{hc}{1\text{ m}}$$

This result is precisely the equality between the second and third terms of the *Fundamental Equation*. This demonstrates that the equation is not just a numerical curiosity but emerges naturally from a core physical principle: *classical equipartition, once dressed by the dissipative nature of the quantum vacuum*.

## Part III: The Geometric Origin of Mass, Charge, and Fundamental Interactions

### X. THE EMERGENT NATURE OF MASS

In this section, we establish the interpretation of mass not as a fundamental, intrinsic property of matter, but as an emergent phenomenon arising from the elastic and quantum nature of the vacuum.

#### A. Mass as an Elastic Response: A Constitutive Law

Our starting point is to equate the two most fundamental expressions for force: Newton's second law, which defines inertial force, and Hooke's law, which describes an elastic restoring force.

$$\vec{F} = m \cdot \vec{a} \quad (\text{Newton's Second Law}) \quad (58)$$

$$\vec{F} = -k\vec{x} \quad (\text{Hooke's Law}) \quad (59)$$

By equating them, we link the inertia of a body ( $m$ ) to the elastic properties ( $k$ ) of the system it interacts with:

$$m \cdot \vec{a} = -k \cdot \vec{x}$$

Solving for mass, we obtain an expression that defines it in terms of elastic response:

$$m = -k \cdot \frac{x}{a}$$

The term  $x/a$  has the physical dimensions of time squared,  $[T^2]$ . Within our unified dimensional framework (where  $[M] \equiv [L] \equiv [T]$ ), we note that this is precisely the dimension of **Action**,  $S$ :

$$[S] = [E \cdot T] \equiv [L \cdot L] = [L^2] \equiv [T^2]$$

This profound dimensional equivalence allows us to propose a fundamental relationship defining mass. By identifying the ratio  $x/a$  with the Action  $S$ , we arrive at a *constitutive law for mass*:

$$\boxed{m \equiv -kS} \quad (60)$$

In this view, mass is not a primitive quantity but a measure of the vacuum's elastic response. This relation has a profound physical interpretation:

- $S$  represents the Action, which quantifies the dynamic deformation of a region of space-time.
- $k$ , with dimensions of frequency ( $[k] = [m]/[S] \equiv [T]/[T^2] = [T^{-1}]$ ), represents the **natural frequency of the vacuum oscillator mode** being excited.

Thus, the mass of a particle is directly proportional to the natural frequency of the vacuum mode it perturbs. A higher mass corresponds to an excitation of a higher-frequency (i.e., "stiffer") mode of the vacuum lattice. The negative sign in the original derivation reflects the restoring, inertial nature of this response.

This formulation expresses that mass is a manifestation of the vacuum's resistance to deformation. It coheres with our later identification of gravitational stiffness via the damping tensor  $\zeta_{\mu\nu}$  and can be generalized as a contraction between the vacuum's internal stress and strain tensors,  $m \sim \zeta_{\mu\nu} \mathcal{G}^{\mu\nu}$ .

#### B. The Mass-Energy of a Vacuum Excitation

Having established the general principle  $m \equiv -kS$ , we now apply it to a single quantum excitation of the vacuum. This allows us to derive the familiar expression for the energy of a photon from a completely new perspective.

In our framework, the elastic stiffness  $k$  of a vacuum oscillator mode has dimensions of frequency. We can therefore identify  $k$  with the quantum of angular frequency,  $k = c/\lambda$ . Simultaneously, the smallest unit of action  $S$  for any quantum process is Planck's constant,  $S \rightarrow h$ .

Substituting these quantum equivalents into our constitutive law for mass (Eq. 60):

$$m \equiv -k \cdot S \quad \longrightarrow \quad m \equiv \frac{hc}{\lambda} \quad (61)$$

This derivation is remarkable. It shows that the mass-energy equivalence for a photon (or other quantum excitations) is not just a postulate of

quantum mechanics, but a direct consequence of interpreting mass as the elastic response of a quantized vacuum, and re-discovers Einstein's mass-energy equivalence from a fundamentally new perspective.

### *Analogy with the Higgs Mechanism*

It is worth noting the conceptual parallel between this model of emergent mass and the *Higgs mechanism* of the Standard Model. In both frameworks, mass is not an intrinsic property of a particle but arises from its interaction with a background field that permeates all of space.

- In the **Standard Model**, a particle acquires mass by interacting with the scalar Higgs field; the strength of this interaction determines its mass.
- In our **Quantum Oscillator Lattice** model, a particle's mass is a measure of the inertial resistance it encounters from the elastic spacetime fabric; the mass is determined by the natural frequency ( $k$ ) of the vacuum mode it excites.

In this sense, the vacuum's elastic modulus  $k$  plays a role analogous to the Higgs field's vacuum expectation value (VEV). This parallel suggests that our elastic-geometric approach provides an alternative, and perhaps deeper, physical picture for the origin of inertia. Indeed, rather than being merely analogous, it is plausible that the Standard Model Higgs field is not fundamental, but instead emerges as an effective excitation mode of the unified field  $\mathcal{G}_{\mu\nu}$ . Specifically, the scalar Higgs field could correspond to the trace of the deformation tensor,  $\mathcal{G} = g^{\mu\nu}\mathcal{G}_{\mu\nu}$ , or to its purely scalar component  $\mathcal{G}_{00}$  in the low-energy limit. In this scenario, the Higgs mechanism would be a consequence of the scalar "elasticity" of spacetime, while the other components of  $\mathcal{G}_{\mu\nu}$  (torsional and shear) would give rise to the gauge and gravitational interactions, providing a common geometric origin for both inertia and forces.

## XI. THE NATURE OF THE GRAVITATIONAL CONSTANT $G$

### A. Static Origin: $G$ as the reciprocal of electromagnetic stiffness

The structural equivalence between mass and charge in our unified framework,  $[M] \equiv [Q]$ , implies a deep connection between the gravitational

and electrostatic coupling constants. This is immediately apparent in the structural similarity of their respective field equations:

Field Law	Electrostatics	Gravitation
Gauss' law (diff.)	$\nabla \vec{E} = 4\pi K_e \rho_e$	$\nabla \vec{g} = -4\pi G \rho_m$
Radial field	$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$	$g = -G \frac{M}{r^2}$
Potential field	$\phi_E = \frac{Q}{4\pi\epsilon_0 r}$	$\phi_G = -\frac{GM}{r}$

These expressions differ only in sign (attractive vs. repulsive) and in their coupling constants:  $\epsilon_0^{-1}$  and  $G$ , respectively. If mass and charge are interpreted as equivalent structural sources of field deformation, then the vacuum constants  $\epsilon_0$  and  $G$  must also be manifestations of the same underlying property of space.

We interpret this duality through the lens of vacuum elasticity. The Coulomb constant,  $K_e$ , being very large, represents the vacuum's immense *stiffness* or resistance to transverse (electrostatic) deformation. Conversely, the gravitational constant,  $G$ , being very small, represents the vacuum's extreme *compliance* (or softness) to longitudinal (gravitational) deformation.

In physics, stiffness and compliance are naturally reciprocal concepts. We therefore propose that the most fundamental relationship between the two constants is one of inversion, scaled by a dimensionless geometric factor,  $\mathcal{C}_{geom}$ :

$$G = \mathcal{C}_{geom} \cdot \frac{1}{K_e} = \mathcal{C}_{geom} \cdot 4\pi\epsilon_0 \quad (62)$$

Our task is to determine the value of  $\mathcal{C}_{geom}$  from first principles. We can find this factor by analyzing the self-energy required to assemble a uniform sphere of charge, a classic problem in electrostatics [32]. Consider the energy  $U$  required to assemble a sphere of charge with a uniform charge density, also known as the self-energy of some sphere [32], with elementary charge  $e$  and radius  $r$ , which can be expressed [32] as

$$U_{sphere} = \frac{3}{5} \cdot \frac{e^2}{4\pi\epsilon_0 r} \quad (63)$$

The total energy  $U$  in the system is related to its capacitance  $C$  and the potential  $V$  by:

$$U = CV^2$$

The potential (voltage)  $V$  at the surface of the sphere [33] is:

$$V = \frac{1}{4\pi\epsilon_0} \frac{e}{r}$$

We can express  $C$  in terms of the self-energy  $U$  and the potential  $V$  as:

$$C = \frac{U}{V^2}$$

Substituting the expressions for  $U$  and  $V$ , and operating, we have:

$$\begin{aligned} C &= \frac{U}{V^2} = \frac{\frac{3}{5} \cdot \frac{e^2}{4\pi\epsilon_0 r}}{\left(\frac{1}{4\pi\epsilon_0} \frac{e}{r}\right)^2} \\ &= \frac{3}{5} 4\pi\epsilon_0 r \rightarrow \frac{C}{r} = \frac{3}{5} 4\pi\epsilon_0 \end{aligned} \quad (64)$$

Within our framework,  $[C] = [L] = [T]$  and  $\epsilon_0$  becomes dimensionless, so both sides of the equation become dimensionless (and thus, dimensionally consistent).

Note that, numerically, with the current accepted value for  $\epsilon_0$  [34], we have that

$$\frac{3}{5} \cdot 4\pi\epsilon_0 \approx 6.6759 \times 10^{-11}$$

Which is indeed pretty close to the established value of the gravitational constant  $G$  [35]. Therefore, and as the dimensionless factor  $\frac{3}{5}$  arises purely from the geometry of integrating the energy density over a spherical volume, we identify this value as the geometric constant of proportionality we seek:

$$C_{geom} = \frac{3}{5}$$

Substituting this geometric factor back into our reciprocity principle (Eq. 62), we derive the definitive expression for  $G$ :

$$\boxed{G \equiv \left(\frac{3}{5}\right) \frac{1}{K_e} = \left(\frac{3}{5}\right) 4\pi\epsilon_0} \quad (65)$$

which confirms the proposed reciprocity principle: the vacuum's *longitudinal compliance* (gravitational softness) is inversely proportional to its *transverse stiffness* (electromagnetic tension). Put differently, the vacuum's ability to resist gravitational deformation is weakest precisely because it is most rigid electromagnetically. The stronger the electric field that the vacuum can sustain, the weaker the gravitational interaction that can emerge from the same medium.

This duality highlights a profound symmetry: *gravity and electromagnetism are orthogonal projections of the same underlying field elasticity*. The vacuum's ability to deform under longitudinal (gravitational) excitation is far more limited than under transverse (electromagnetic) excitation.

In other words, the vacuum is extremely stiff in response to mass-like perturbations (as encoded in the smallness of  $G$ ), and relatively compliant to charge-like perturbations (as seen in the largeness of  $K_e$ ).

This not only reinforces the modal equivalence described previously but gives it a new dimension: the vacuum acts as a geometric impedance surface, whose tension and compliance balance across regimes to define the apparent strengths of fundamental forces. The interpretation  $G \propto \epsilon_0 \sim 1/K_e$  thus follows naturally as a structural necessity within this framework.

## B. A Scale-Dependent Gravitational Coupling Framework

We have previously shown how the gravitational constant can be derived from the self-energy of a uniformly dense sphere, resulting in:

$$G = \frac{3}{5} 4\pi\epsilon_0$$

On the other hand, consider the electrostatic energy stored in a charged spherical conductor of capacitance  $C$  and charge  $Q$  ([32]), given by:

$$U_{\text{capacitor}} = \frac{1}{2} \frac{Q^2}{C} \quad (66)$$

For a sphere of radius  $r$ , the capacitance is  $C_{\text{sphere}} = 4\pi\epsilon_0 r$ , so the stored energy becomes:

$$U_{\text{Glob}} = \frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r}$$

This expression lacks the geometric self-energy term present in the self-energy case. If we derive the capacitance from the stored energy  $U_{\text{Glob}}$  and potential  $V = \frac{1}{4\pi\epsilon_0} \frac{e}{r}$  we find:

$$C = \frac{U_{\text{Glob}}}{V^2} = 2\pi\epsilon_0 r \Rightarrow \frac{C}{r} = 2\pi\epsilon_0$$

The derivation of  $G$  from the self-energy of a dense sphere (65) implicitly models a regime dominated by *localized, gravitationally bound systems* where non-linear self-interaction effects are maximal. This is an excellent model for local structures such as galaxies and clusters. On *cosmological scales*, however, the Universe is observed to be highly homogeneous and isotropic. In such a *quasi-linear regime*, the strong gravitational nonlinearities that characterize local clumps are expected to average out and become negligible. We therefore hypothesize that the effective gravitational coupling on these vast scales converges to a baseline state where these self-interaction terms

are suppressed. The electrostatic case of an ideal spherical conductor, which lacks the volumetric self-energy factor, provides a compelling structural analogue for this global, quasi-linear regime.

This leads to a dual effective gravitational coupling framework:

- **Local Scales:** Characterized by significant inhomogeneity (e.g., galaxies and clusters), gravitational dynamics are modeled with an effective coupling

$$G_{\text{Loc}} = \frac{3}{5}4\pi\epsilon_0$$

This value corresponds to the full self-interaction contribution and is identified with the standard Newtonian constant  $G_N$ .

- **Global Scales:** At cosmological scales, where the Universe is approximately homogeneous and isotropic, gravitational dynamics are governed by a reduced effective coupling

$$G_{\text{Glob}} = 2\pi\epsilon_0$$

This dual effective gravitational coupling framework interprets the non-linearities of gravity—prominent in clumpy, small-scale environments—as contributing an additional self-energy component to the effective gravitational coupling, resulting in  $G_{\text{Loc}} \equiv G_N$ . However, on very large, homogeneous scales, these effects average out such that the effective coupling approaches a baseline value. In our electrostatic analogy, this baseline corresponds to the case of a spherical conductor without the geometric self-energy term, yielding  $G_{\text{Glob}}$ .

This scale dependence on some effective gravitational constant  $G_{\text{eff}}(z)$  is proposed as the underlying physical mechanism unifying the explanation of several major cosmological puzzles, including the *Hubble tension*, the *dark sector* phenomena, and the *growth of structure tension*, as will be detailed in Section XXIV B.

### C. Dynamic Origin: $G$ as a Quadratically-Damped Vacuum Response

Having proposed the static nature of  $G$  as a measure of compliance, we now investigate its dynamic origin. We propose that gravity is not a primary interaction, but an *effective, second-order response* of the vacuum to the presence of mass-energy. The primary vacuum interaction is electromagnetic, with an intrinsic transverse

stiffness quantified by  $\mu_0$ . Gravity emerges as a residual effect of this primary interaction, quadratically suppressed by the vacuum's own damping mechanism.

The fundamental damping ratio of the vacuum is  $\zeta = \alpha$ . As a second-order phenomenon, we expect the gravitational coupling  $G$  to be proportional to the primary stiffness,  $\mu_0$ , scaled by the square of the damping factor:

$$G \propto \mu_0 \cdot \zeta^2 = \mu_0 \cdot \alpha^2$$

And thus we can propose a first-order dynamic expression for  $G$ :

$$\boxed{G \equiv \mu_0 \alpha^2} \quad (67)$$

This result, derived from a physical model of quadratic damping, is validated by its remarkable numerical accuracy:

$$G \approx (4\pi \times 10^{-7}) \cdot (0.007297)^2 \approx 6.69 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

This provides strong support for the interpretation of gravity as a residual, quadratically-damped manifestation of the underlying electromagnetic properties of the vacuum.

#### *The Structural Damping Tensor and the Constitutive Law of Gravity*

The above result motivates a more formal, tensorial description. We can promote the scalar damping ratio  $\zeta = \alpha$  to the norm of a rank-2 symmetric tensor,  $\zeta_{\mu\nu}$ , which we define as the *structural damping tensor* of the vacuum. This tensor encodes the vacuum's anisotropic dissipative response to perturbations. The effective damping strength is its scalar contraction:  $\zeta^2 = \zeta_{\mu\nu} \zeta^{\mu\nu}$ .

In this context,  $G = \mu_0 \cdot \zeta^2$  admits a natural interpretation within our unified field framework: it is the constitutive law that relates the vacuum's transverse rigidity coupling  $\mu_0 c^2$  to its longitudinal stiffness  $G$  through the internal damping response  $\zeta^2 = \zeta_{\mu\nu} \zeta^{\mu\nu}$ . In direct analogy with classical elasticity theory, where the stored elastic energy takes the form  $E_{\text{pot}} = \frac{1}{2} \sigma_{\mu\nu} \epsilon^{\mu\nu}$ , we may view  $\zeta_{\mu\nu}$  as the internal stress tensor of the vacuum, induced by the strain field  $\mathcal{G}_{\mu\nu}$ . In this picture, gravity emerges as the quadratic response of the vacuum's elastic-dissipative lattice to coherent longitudinal deformations induced by mass-energy excitations.

Thus, the gravitational constant  $G$  is not

fundamental, but derived: it encodes the effective elastic modulus of the vacuum along longitudinal modes, just as  $\mu_0$  encodes transverse response. The relation  $G = \mu_0 \cdot \zeta^2$  then completes the analogy with continuum mechanics, anchoring the gravitational interaction in the internal modal geometry and dissipative structure of the vacuum field.

#### D. The Global Gravitational Constant as the Vacuum's Maximum Current Amplitude

##### *Derivation*

Our framework reveals a remarkable and profound connection between the cosmological scale of gravity and the microscopic dynamics of a single vacuum oscillator. This link is established by identifying the global gravitational constant,  $G_{\text{Glob}}$ , with the maximum current amplitude,  $I_{\text{max}}$ , that a fundamental vacuum "cell" can sustain.

In any simple harmonic oscillator, the maximum current is the product of its charge amplitude ( $Q_0$ ) and its natural angular frequency ( $\omega$ ). Within our model, we identify these fundamental parameters as the elementary charge,  $e$ , and the reference angular frequency,  $\omega = c/(1 \text{ m})$ , respectively. The maximum current amplitude of the vacuum oscillator is therefore:

$$I_{\text{max}} = e \cdot \omega = e \cdot \frac{c}{1 \text{ m}} \quad (68)$$

which is the same that we have derived in (37). We can now substitute the expression for the elementary charge derived from our framework's first principles,  $e \equiv \frac{2\alpha \cdot 1 \text{ m}}{c^2}$ :

$$I_{\text{max}} \equiv \left( \frac{2\alpha \cdot 1 \text{ m}}{c^2} \right) \cdot \left( \frac{c}{1 \text{ m}} \right) = \frac{2\alpha}{c}$$

This gives us a deep physical meaning for the term  $\frac{2\alpha}{c}$ : it represents the peak dynamic current of a fundamental vacuum excitation.

The crucial insight comes from deriving an independent expression for  $G_{\text{Glob}} = 2\pi\varepsilon_0$  directly from the fine-structure constant's definition. The formula for  $\alpha$  is:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

We can rearrange this to solve for  $G_{\text{Glob}}$ :

$$G_{\text{Glob}} = 2\pi\varepsilon_0 = \frac{e^2}{2\alpha\hbar c}$$

Now, we substitute the expressions for the elementary charge ( $e \equiv \frac{2\alpha \cdot 1 \text{ m}}{c^2}$ ) and the reduced Planck

constant ( $\hbar \equiv \frac{1 \text{ m}^2}{c^4}$ ) that emerge from our framework:

$$\begin{aligned} G_{\text{Glob}} &\equiv \frac{\left( \frac{2\alpha \cdot 1 \text{ m}}{c^2} \right)^2}{2\alpha \left( \frac{1 \text{ m}^2}{c^4} \right) c} \\ &= \frac{\frac{4\alpha^2}{c^4}}{\frac{2\alpha}{c^3}} = \frac{4\alpha^2}{c^4} \cdot \frac{c^3}{2\alpha} \\ &= \frac{2\alpha}{c} \end{aligned} \quad (69)$$

By comparing these two results, we arrive at a stunningly simple and powerful identity:

$$\boxed{G_{\text{Glob}} \equiv I_{\text{max}}} \quad (70)$$

This equation establishes a direct bridge between the macroscopic and microscopic realms. It asserts that the parameter governing the large-scale gravitational dynamics of a homogeneous universe ( $G_{\text{Glob}}$ ) is numerically identical in a first-order approximation to the maximum current amplitude of a single quantum oscillator of the vacuum.

The weakness of gravity, from this perspective, is no longer an arbitrary accident of nature. It is a direct reflection of the tiny current that a fundamental vacuum excitation can carry. This result provides a powerful physical intuition, unifying cosmology and quantum dynamics not through energy densities, but through a fundamental, quantized current. It serves as one of the most compelling self-consistency checks of the entire theoretical framework.

##### *Consistency check: the Inductive Origin of the Vacuum Energy Density*

An alternative and powerful derivation of the vacuum energy density,  $\rho_{\text{vac}}$ , emerges from the vacuum's fundamental inductive properties. This approach is grounded in two core principles of the established framework: the characteristic vacuum inductance,  $L_{\text{vac}} = \mu_0 \cdot 1 \text{ m}$  (IV B), and the maximum current amplitude that a fundamental vacuum oscillator can sustain,  $I_{\text{max}} \equiv 2\alpha/c$ , as derived in Eq. 69.

While a classical oscillator's energy is typically time-averaged, a fundamental quantum mode is a persistent, coupled state. We posit that its total energy is the sum of the peak potentials of its magnetic and electric modes. Assuming equipartition between these modes, the total peak energy,  $E_{\text{peak}}$ , is twice the peak magnetic energy:

$$E_{\text{peak}} = 2 \cdot L_{\text{vac}} I_{\text{max}}^2 = 2(\mu_0 \cdot 1 \text{ m}) \left( \frac{2\alpha}{c} \right)^2 \quad (71)$$

Within our unified framework, where energy and mass are dimensionally equivalent via  $[M] \equiv [L] \equiv [T]$ , this expression for  $E_{\text{peak}}$  yields a quantity with the physical dimensions of mass. We interpret this value as the fundamental mass-energy contained within the characteristic unit volume of the vacuum,  $V_{\text{ref}} = (1 \text{ m})^3$ . The vacuum mass density,  $\rho_{\text{vac}}$ , is therefore defined as this peak energy distributed over the reference volume. The numerical evaluation yields:

$$\begin{aligned} \rho_{\text{vac}} &= \frac{E_{\text{peak}}}{V_{\text{ref}}} = \frac{2(\mu_0 \cdot 1 \text{ m}) \left(\frac{2\alpha}{c}\right)^2}{(1 \text{ m})^3} \\ &\approx 5.956 \times 10^{-27} \text{ kg/m}^3 \end{aligned} \quad (72)$$

This result is in excellent agreement with the value measured by the Planck Collaboration in 2015 [18], providing strong quantitative support for the interpretation of vacuum energy as an emergent property of the vacuum's peak inductive and electromagnetic response.

### E. Unified Interpretation of the Fundamental Forces

Having derived the nature of the fundamental constants, we can now reinterpret the force laws they govern, revealing that they are not disparate laws, but complementary modal projections of a single underlying mechanism: *momentum exchange through the elastic and dissipative vacuum*.

#### 1. Newton's and Coulomb's Laws as Damped Momentum Transfer

From the previously derived first-order expressions, such as  $G \equiv \mu_0 \cdot \alpha^2$  and  $k_B \equiv \mu_0/c^2$ , it follows that:

$$G \equiv k_B \cdot \alpha^2 \cdot c^2$$

Using this expression, we can rewrite Newton's law to reveal its thermo-entropic nature. This force is mediated by the exchange of *damped, longitudinal momentum*, and expressed in terms of damped relativistic momenta, where  $\zeta = \alpha$  is the norm of the structural damping tensor:

$$F_g \equiv k_B \cdot \frac{(Mc \cdot \zeta) \cdot (mc \cdot \zeta)}{r^2} \quad (73)$$

Thus, the terms in the numerator correspond to effective, damped relativistic momenta, whose propagation is modulated by the damping structure encoded in  $\zeta_{\mu\nu}$ . Gravity emerges as the effective resistance to the coherent alignment of projected momenta through the vacuum's dissipative tensorial geometry.

By symmetry, the Coulomb force can be reformulated as the exchange of *undamped, transverse momentum*:

$$F_e = K_e \cdot \frac{Q_1 Q_2}{r^2} = \frac{\mu_0 \cdot c^2}{4\pi} \cdot \frac{Q_1 Q_2}{r^2} = \mu_0 \cdot \frac{(Q_1 c)(Q_2 c)}{4\pi r^2} \quad (74)$$

This structure mirrors the gravitational expression, with  $Qc$  playing the role of transverse modal momentum and the vacuum magnetic permeability  $\mu_0$  acting as the transverse field stiffness. Thus, both Newton's and Coulomb's laws appear as complementary modal projections of the same unified vacuum tensorial response.

#### 2. Physical Interpretation: the Structural Differences

The distinct mathematical forms of Eqs. 73 and 74 are not accidental; they reflect profound differences in the physical nature of the momentum exchange. Comparing the rewritten laws for the gravitational (Eq. 73) and Coulomb (Eq. 74) forces reveals two fundamental structural differences that provide deep insight into their distinct physical natures.

*a. 1. The Selective Role of Damping.* A crucial difference is the explicit presence of the damping factor,  $\zeta$ , in the gravitational force, while it is absent in the static Coulomb force. This does not imply that electromagnetism is an undamped phenomenon—indeed, we have argued that the propagation of light is damped. Rather, it reveals the fundamental character of each interaction:

- The **Coulomb force** is a *conservative interaction* between static charges. Its formulation does not involve dissipation. The dissipative effects of electromagnetism arise in dynamic phenomena, such as radiation.
- The **gravitational force**, when expressed in this thermo-entropic form, is revealed to be an inherently *dissipative and entropic interaction*. It is not a static potential force in the classical sense, but the result of momentum exchange through a dissipative medium. Therefore, it *must* explicitly include the damping factor  $\zeta$  as part of its fundamental definition.

This suggests that damping is a key feature that distinguishes the entropic (longitudinal) modes of the vacuum from the conservative (transverse) static modes.

*b. 2. The Geometry of Propagation ( $r^2$  vs.  $4\pi r^2$ ).* The second key difference lies in the geometry of the denominator.

- The **Coulomb force** includes the factor  $4\pi r^2$ , the surface area of a sphere. This reflects the isotropic, wave-like nature of the interaction, where the influence of a point charge propagates outwards uniformly in all directions, spreading over a spherical surface. This is characteristic of a **transverse wave**.
- The re-expressed **gravitational force**, by contrast, lacks the  $4\pi$  solid angle factor. Its  $r^2$  dependence represents a direct interaction between two points. This supports the interpretation of a **longitudinal interaction**, where the force acts as a direct "pressure" or "tension" along the line connecting the two masses, rather than as a field radiating spherically.

In summary, both interactions reflect momentum exchange across a structured medium, but what differs is the symmetry of the exchange. The immense difference in their strengths arises not from arbitrarily different coupling constants, but from the physical properties of the vacuum itself:

- The gravitational force emerges as a highly suppressed, entropy-weighted longitudinal momentum flow. Gravity is weak not because its coupling is small, but because the vacuum is extremely rigid against this type of compressional deformation, and the Boltzmann constant  $k_B$  quantifies the high entropic cost required.
- The electromagnetic force, in contrast, reflects a much more efficient, transversely mediated momentum exchange, amplified by the vacuum's comparatively soft resistance to shear-like deformations, a process governed by  $\mu_0$ .

## XII. SYNTHESIS: UNIFYING THE VACUUM'S ELASTIC AND QUANTUM PROPERTIES

We have now established a complete, two-scale framework for the gravitational constant derived from both static and dynamic principles. The ultimate test of this framework is its internal consistency. We will now demonstrate that the relationships derived are not only mutually compatible but also lead to profound connections between the vacuum's elastic, electromagnetic, and quantum characteristics.

### A. A Consistency Condition for the Vacuum

We have derived the gravitational constant  $G$  from two independent perspectives, grounded in the vacuum's elastic properties:

- **Static Origin:** Based on a principle of reciprocity, where  $G$  is the geometrically-scaled compliance of the vacuum:

$$G = \left(\frac{3}{5}\right) 4\pi\epsilon_0$$

- **Dynamic Origin:** Based on a model of quadratic damping, where  $G$  is the primary vacuum stiffness, quadratically suppressed by dissipation:

$$G = \mu_0\alpha^2$$

For this framework to be self-consistent, these two expressions for  $G$  must be equal. Equating them reveals a profound **consistency condition** that the vacuum's properties must obey:

$$\boxed{\left(\frac{3}{5}\right) 4\pi\epsilon_0 \equiv \mu_0\alpha^2} \quad (75)$$

This equation is a powerful prediction of the theory. Using the relation  $Z_0^2 = \frac{\mu_0}{\epsilon_0}$ , we can rewrite this condition as:

$$Z_0 \equiv \frac{\sqrt{\frac{3}{5}4\pi}}{\alpha} \approx 376.28 \Omega \quad (76)$$

where one can identify the second-order term and write the more exact expression

$$Z_0 \equiv \frac{\sqrt{\frac{3}{5}4\pi}}{\alpha} \left(1 + \frac{\alpha}{2\pi} + \dots\right) \approx 376.73 \Omega$$

Another direct and powerful corollary is a relationship between  $\alpha$ ,  $G$ , and  $\mu_0$ . Rearranging Eq. (75) yields:

$$\alpha \equiv \sqrt{\frac{G}{\mu_0}} \quad (77)$$

This simple expression confirms that  $\alpha$  is not an independent constant, but a *geometric attenuation coefficient* derived from the ratio of the vacuum's longitudinal and transverse stiffnesses, and elevates the fine-structure constant from a mere electromagnetic coupling parameter to a *universal measure of vacuum damping topology*, governing how all interactions manifest.

### B. The quality factor as a Quantum-Electrodynamic Property

From the definition of the fine-structure constant, we have:

$$2\alpha = \frac{e^2}{2\pi\epsilon_0\hbar c}$$

Using the key consistency condition derived from our model  $G_{Glob} \equiv 2\pi\epsilon_0 \equiv 2\alpha/c$  (69), and substituting this into the denominator gives:

$$2\alpha \equiv \frac{e^2}{\left(\frac{2\alpha}{c}\right)\hbar c} = \frac{e^2}{2\alpha\hbar} \rightarrow \boxed{4\alpha^2 \equiv \frac{1}{Q^2} \equiv \frac{e^2}{\hbar}} \quad (78)$$

This remarkable result implies that the ability of a channel to conduct charge is determined by the dissipative properties of the spacetime in which it exists.

The Quantum Hall Effect (QHE) shows that resistance is quantized in units of the **von Klitzing constant**,  $R_K$ :

$$R_K = \frac{h}{e^2} = 2\pi \frac{\hbar}{e^2} \equiv 2\pi Q^2$$

This implies that the quantized resistance measured in a 2D electron gas is not an emergent property of the material system alone, but rather a direct manifestation of the vacuum's intrinsic damping. The electrons in a QHE system are, in effect, probing the fundamental dissipative structure of spacetime itself.

### C. A Unified Emergence Mechanism for Gravity and Charge

The internal consistency of our framework reveals a profound structural symmetry between the origin of the local gravitational constant,  $G_{Loc}$ , and the squared elementary charge,  $e^2$ . As derived in Section 67:

$$G_{Loc} \equiv \mu_0 \cdot \zeta^2 \quad (79)$$

We now derive an analogous expression for the squared elementary charge. Starting from our derived first-order equivalence  $e \equiv \frac{2\alpha \cdot 1\text{m}}{c^2}$ , squaring both sides yields:

$$e^2 \equiv 4\alpha^2 \cdot \frac{(1\text{m})^2}{c^4} \rightarrow \frac{1}{4}e^2 \equiv \hbar \cdot \zeta^2$$

The parallel between the two results is striking and reveals a universal mechanism. Both gravity and charge are emergent properties determined by the same squared damping factor,  $\zeta^2 \equiv \alpha^2$ , acting as a "universal dissipative filter." The fundamental

interactions and charges of our universe become "echoes" of more primordial vacuum properties ( $\mu_0$  and  $\hbar$ ), all filtered through the same dissipative process ( $\zeta^2$ ). The difference in their nature and magnitude arises from their distinct origins:

- **Gravity** emerges from the *classical, elastic* properties of the vacuum ( $\mu_0$ ).
- **Charge** emerges from the *fundamental, quantum* properties of the vacuum ( $\hbar$ ), and as an effective electromagnetic field coupling, it adopts the canonical normalization factor of  $\frac{1}{4}$ .

This shared emergence mechanism is one of the most powerful pieces of evidence for the coherence of the proposed framework, suggesting a deep unity in the principles that govern the cosmos.

### D. A Bridge to General Relativity: An Ohm's Law for the Vacuum

Additionally, our framework provides a remarkable consistency check with General Relativity. Substituting  $G \equiv \mu_0 \cdot \alpha^2$  one can check that

$$G \cdot c \equiv \mu_0 \cdot c \cdot \alpha^2 = Z_0 \cdot \alpha^2 \approx \frac{1}{50.13} \Omega \quad (80)$$

The above implies that the product  $G \cdot c$  defines a fundamental 'resistive-like' constant of the vacuum, which we denote as the *natural resistance*  $X_N$ . The relationship  $G \equiv \frac{1}{16\pi c}$  can be interpreted as an Ohm's Law for the vacuum,  $V = I \cdot R$ , where:

- **Potential ( $V$ ):** The gravitational constant  $G$ , within our dimensionally collapsed framework, acquires the role of a fundamental potential or electromotive force, as  $G \equiv \mu_0 \cdot \zeta^2$  and  $[\mu_0] = [V]$  (19).
- **Current ( $I$ ):** The term  $1/c$  represents the natural, scaled current of the vacuum for the thermo-entropic mode (VC).
- **Resistance ( $R$ ):** This implies that the vacuum possesses an intrinsic, dimensionless resistance  $R_{vac} = 1/(16\pi)$ .

This vacuum resistance is not an arbitrary number, but can be derived from first principles by decomposing it into two meaningful factors:

$$R_{vac} = \frac{1}{16\pi} = \frac{1}{4} \cdot \frac{1}{4\pi} \quad (81)$$

The two terms have clear physical origins:

1. **The Geometric Resistance ( $1/4\pi$ ):** This factor arises directly from the Green's function of the 3D Laplacian operator (XXB).

It represents the fundamental geometric impedance of three-dimensional space, quantifying how the influence of a point source is diluted as it spreads over a spherical solid angle.

## 2. The Canonical coupling factor (1/4):

This factor is the canonical normalization constant required for any standard gauge field theory, as seen in the electromagnetic Lagrangian  $\mathcal{L}_{EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ . Its presence here suggests that gravity, as an *emergent effective field*, must inherit the canonical normalization of the underlying field theory framework.

Therefore, the relationship we propose is not a numerical coincidence, but a profound statement about the structure of the vacuum. The equation

$$G \equiv \frac{1}{c} \cdot \left( \frac{1}{4} \cdot \frac{1}{4\pi} \right) \implies c \equiv \frac{1}{16\pi G} \quad (82)$$

serves as a fundamental "bridge" between our elastic vacuum model and the geometric formulation of General Relativity. It shows that the pre-factor in the Einstein-Hilbert action,

$$S_{EH} = \frac{c^4}{16\pi G} \int R \sqrt{-g} d^4x,$$

is not a random assortment of constants. Instead, the term  $c^4/(16\pi G)$  can be seen as the "stiffness" of spacetime against curvature, emerging directly from the interplay between the vacuum's fundamental current, its geometric properties, and the canonical structure of field theory (1/4).

### E. The Impedance of Free Space as a Damped Geometric Resistance

Note that we can rewrite eq. 80 just substituting  $\zeta \equiv \alpha$  and  $\frac{1}{50.13} \equiv \frac{1}{16\pi}$ , to have that:

$$\boxed{Z_0 \cdot \zeta^2 \equiv \frac{1}{4} \cdot \frac{1}{4\pi}} \quad (83)$$

where the geometric interpretation of the terms in the left has already been exposed (XIID). Thus, the physical impedance of the vacuum,  $Z_0$ , can be described as the fundamental geometric resistance modulated by the vacuum's own damping factor,  $\zeta = \alpha$ . This equation unifies geometry, quantum dissipation, and electromagnetism in a single, elegant expression.

### F. First-Principles Prediction of the Fine-Structure Constant

A profound consequence of the vacuum's self-consistency is that the value of the fine-structure

constant,  $\alpha$ , becomes uniquely determined by the geometric and topological constraints of the framework. In the preceding sub-subsections, we derived two independent, constitutive laws for the vacuum impedance:

1. From the consistency between the static and dynamic origins of the gravitational constant  $G$ , we obtained (Eq. XII A):

$$Z_0 \equiv \frac{\sqrt{\frac{3}{5}4\pi}}{\alpha} \quad (84)$$

2. From the interpretation of  $Z_0$  as a damped geometric resistance, we derived (Eq. 83):

$$Z_0 \equiv \frac{1}{4} \frac{1}{4\pi\zeta^2} = \frac{1}{16\pi\alpha^2} \quad (85)$$

For the theoretical framework to be internally consistent, these two expressions for  $Z_0$  must be equal. Equating Eq. 84 and Eq. 85 forces a single, unique value for the fine-structure constant:

$$\frac{\sqrt{\frac{3}{5}4\pi}}{\alpha} \equiv \frac{1}{16\pi\alpha^2} \quad (86)$$

Solving for  $\alpha$  yields:

$$\alpha \equiv \frac{1}{16\pi\sqrt{\frac{3}{5}4\pi}} \quad (87)$$

where one can identify the second-order term and write the more exact expression

$$\alpha \equiv \frac{1}{16\pi\sqrt{\frac{3}{5}4\pi}} \left( 1 + \frac{\alpha}{2\pi} + \dots \right)$$

The first-order predicted value matches the experimentally measured CODATA value of  $\alpha \approx 1/137.036$  with remarkable accuracy, and is highly significant. It shows that the value of the fine-structure constant is not an arbitrary input, but rather the precise value required to harmonize the vacuum's fundamental properties: its geometric impedance arising from the 3D Laplacian ( $\frac{1}{4\pi}$ ), its canonical field structure inherited from gauge theories ( $\frac{1}{4}$ ), and the geometry of its self-interaction energy ( $\sqrt{\frac{3}{5}4\pi}$ ). This elevates the status of  $\alpha$  from a mere electromagnetic coupling parameter to the primary geometric and topological constant of the unified vacuum.

As a final note, for the above interpretation to be consistent, then  $\frac{1}{\sqrt{\frac{3}{5}4\pi}}$  must have dimensions of reciprocal of a resistance. We will uncover and check that this is indeed the case in (XII G).

### G. Impedance vs. Dissipative Resistance and the quality factor $Q$

The robustness of this theoretical framework can be further tested by examining the consistency of the classical RLC oscillator analogy, which has served as a powerful conceptual guide. Consider the standard formula for the quality factor of a series RLC circuit at resonance:

$$Q = \frac{\omega_0 L}{R} \quad (88)$$

Within our framework, we have already established the following vacuum parameters:

- The vacuum quality factor, derived from quantum conductance, is  $Q = 1/(2\alpha)$  (51).
- The characteristic inductive term is  $\omega_0 L = (\frac{c}{1s})(\mu_0 \cdot 1m)$ . Given the spacetime equivalence where  $1s \equiv 1m$ , this simplifies to  $\omega_0 L = \mu_0 c$ , which is precisely the definition of the vacuum's wave impedance,  $Z_0$ .

Substituting these established quantities into the classical Q-factor formula allows us to solve for the effective resistance,  $R_Q$ , that must govern this specific dissipative process:

$$\frac{1}{2\alpha} = \frac{Z_0}{R_Q} \quad (89)$$

This consistency requirement leads directly to a new constitutive law for the vacuum:

$$R_Q = 2\alpha Z_0 \equiv \frac{Z_0}{Q} \equiv 2 \cdot \sqrt{\frac{3}{5}} 4\pi \quad (90)$$

This result is profound. It reveals that the vacuum possesses two distinct, conceptually different resistive properties:

**1. Wave Impedance ( $Z_0$ ):**  $Z_0$  is the vacuum's characteristic impedance to wave propagation. It is a reactive property that governs the ratio of the electric and magnetic field strengths in an electromagnetic wave.

**2. Dissipative Resistance ( $R_Q$ ):** This newly derived quantity,  $R_Q$ , is the vacuum's effective resistance governing the *rate of energy dissipation per cycle* in an oscillation, as quantified by the quality factor. It is a fundamentally dissipative, rather than reactive, property.

Equation 90 provides the explicit relationship between these two properties. It states that the dissipative resistance of the vacuum is its fundamental wave impedance modulated by its own universal quality factor,  $Q$ . This is physically intuitive: the resistance to energy loss

should be proportional to both the resistance to wave propagation and the intrinsic friction of the medium.

Note that using Eq. 90, we can express  $\frac{3}{5}4\pi = \frac{1}{4}R_Q^2$ . Substituting this into the left-hand side of the master consistency condition (Eq. 75) yields:

$$\frac{1}{4}\varepsilon_0 R_Q^2 \equiv \mu_0 \alpha^2 \quad (91)$$

This final expression is a remarkably powerful statement of unification. The dimensional consistency of Eq. 91 holds in both the SI system and the dimensionally-collapsed framework of this theory, further cementing its robustness.

### H. Dynamic Origin of Permittivity from Vacuum Power Principles

Having established the distinct roles of wave impedance ( $Z_0$ ) and dissipative resistance ( $R_Q$ ), we can now propose a dynamic origin for the vacuum's electric permittivity,  $\varepsilon_0$ . We move beyond the static picture of permittivity as a passive capacity to store fields and instead derive it from the vacuum's fundamental power dissipation and resistance properties.

First, we define a *Unitary Vacuum Power*,  $P_{unit}$ , as the power dissipated when the vacuum's intrinsic structural voltage ( $V \equiv \mu_0$ ) is applied across a unitary resistance ( $R = 1\Omega$ ). According to Joule's Law:

$$P_{unit} = \frac{V^2}{R} \equiv \frac{\mu_0^2}{1\Omega} \quad (92)$$

In this framework,  $P_{unit}$  represents the baseline rate of energy transfer or dissipation inherent to the vacuum's primary potential.

Next, we postulate that the vacuum's permittivity,  $\varepsilon_0$ , which represents its compliance or ability to "permit" an electric field, is an emergent property. It arises from this unitary power being modulated by the vacuum's own internal friction. The most natural choice for this friction is the **dissipative resistance**,  $R_Q$ , as it governs energy loss per oscillatory cycle. We therefore propose the following constitutive law:

$$\varepsilon_0 \equiv P_{unit} \cdot R_Q = \frac{\mu_0^2}{1\Omega} R_Q \equiv \mu_0^2 \cdot 2\sqrt{\frac{3}{5}} 4\pi \quad (93)$$

where one can identify the second-order term and write the more exact expression

$$\varepsilon_0 \equiv \left( \mu_0^2 \cdot 2\sqrt{\frac{3}{5}} 4\pi \right) (1 + 2\alpha + \dots)$$

### I. The Gravito-Entropic Power Equivalence

Building upon this last relationship, we can establish a direct link between the vacuum's primary inertial properties and its large-scale cosmological dissipation. Eq. 93 yields a direct equivalence:

$$\frac{1}{4}\mu_0^2 \equiv 2\pi\varepsilon_0 \cdot \alpha \equiv G_{glob} \cdot \zeta \quad (94)$$

This equation describes how the vacuum's capacity to store inertial or reactive energy in its transverse (electromagnetic) modes is intrinsically related to the vacuum's compliance to longitudinal deformation ( $G_{glob}$ ) and its damping factor ( $\zeta$ ). Ultimately, Eq. 94 signifies a profound equilibrium: *the vacuum's capacity to store inertial energy in its transverse mode is perfectly balanced by the power it dissipates in its longitudinal mode.* This relationship links the electromagnetic and gravitational sectors through a fundamental equilibrium.

### J. The Elementary Charge as a high-order stiffness of the Vacuum

An additional insight into the vacuum's fundamental properties arises when substituting the geometric definition of  $\alpha$  from Eq. 87 into the expression  $e \equiv \frac{2\alpha \cdot 1 m}{c^2}$  (IX B):

$$e \equiv \frac{2 \cdot 1 m}{c^2} \left( \frac{1}{16\pi\sqrt{\frac{3}{5}4\pi}} \right) = \frac{1 m}{c^2 \cdot 8\pi\sqrt{\frac{3}{5}4\pi}}$$

Using the fundamental relation  $\mu_0\varepsilon_0 = 1/c^2$  to replace  $c^2$ , and substituting the permittivity  $\varepsilon_0$  from Eq. 93, yields:

$$\begin{aligned} e &\equiv \mu_0 \left( \mu_0^2 \cdot 2\sqrt{\frac{3}{5}4\pi} \right) \cdot \frac{1 m}{8\pi\sqrt{\frac{3}{5}4\pi}} \\ &= \frac{1 m \cdot 2\mu_0^3\sqrt{\frac{3}{5}4\pi}}{8\pi\sqrt{\frac{3}{5}4\pi}} = \frac{\mu_0^3 \cdot 1 m}{4\pi} \rightarrow \boxed{e \equiv \frac{\mu_0^3 \cdot 1 m}{4\pi}} \end{aligned} \quad (95)$$

where one can identify the second-order term and write the more exact expression

$$e \equiv \frac{\mu_0^3 \cdot 1 m}{4\pi} \cdot (1 + 2\alpha + \dots)$$

This result shows how charge is a direct, emergent property of the vacuum itself, determined solely by its most fundamental characteristic—the transverse inertial stiffness  $\mu_0$ —projected through the geometry of three-dimensional space ( $\frac{1}{4\pi}$ ).

The cubic dependence ( $e \propto \mu_0^3$ ) signifies that

charge is a non-linear excitation of the vacuum field. While the vacuum's primary elastic response is linear (as seen in wave propagation), the formation of a stable, quantized charge represents a higher-order, self-interaction phenomenon. It is a measure of the vacuum's capacity to sustain a localized, persistent deformation against its own inertial resistance.

Importantly, this formulation elevates the magnetic permeability  $\mu_0$  to the status of the single, primary constant of the electromagnetic sector. The elementary charge  $e$  and the electric permittivity  $\varepsilon_0$  are both derived from it, establishing a clear hierarchy of fundamental constants.

Finally, equation 95 defines vacuum's capacity to manifest charge as a function of its intrinsic stiffness, contributing a big step into the geometrization of physics by demonstrating that not only forces, but also the sources of those forces, arise from the fabric of spacetime.

### K. The Quantum of Action as an Emergent Property of Vacuum Compliance

Following the derivation of the elementary charge from the vacuum's inertial stiffness ( $\mu_0$ ), we show how the quantum of action,  $\hbar$ , can also be derivable from the vacuum's complementary elastic property: its compliance, or permittivity ( $\varepsilon_0$ ). This demonstration serves to ground not only the sources of forces, but the very granularity of quantum mechanics, in the tangible, elastic properties of the vacuum.

Our starting point is the modal action for the electromagnetic field derived in Eq. 96, which established the dimensionality of action as a spacetime area:

$$\hbar \equiv \frac{1 m^2}{c^4} \quad (96)$$

To express  $\hbar$  in terms of permittivity, we use the fundamental relationship  $c^4 = 1/(\mu_0^2\varepsilon_0^2)$  into Eq. (96), which yields:

$$\hbar = (1 m^2) \cdot (\mu_0^2\varepsilon_0^2)$$

Replacing the inertial term  $\mu_0^2$  with its equivalent expression derived from the vacuum's dissipative properties, as given in Eq. 93, reveals  $\hbar$  as a pure

function of vacuum compliance and geometry:

$$\begin{aligned}\hbar &= (1 \text{ m}^2) \cdot \left( \frac{\varepsilon_0}{2\sqrt{\frac{3}{5}4\pi}} \right) \cdot \varepsilon_0^2 \\ &= \frac{\varepsilon_0^3 \cdot (1 \text{ m}^2)}{2\sqrt{\frac{3}{5}4\pi}}\end{aligned}$$

This leads to a new fundamental expression for the reduced Planck constant:

$$\boxed{\hbar \equiv \frac{\varepsilon_0^3 \cdot (1 \text{ m}^2)}{2\sqrt{\frac{3}{5}4\pi}}} \quad (97)$$

This result provides a compelling physical origin for the quantum of action, reinterpreting it not as an axiom, but as a direct consequence of the vacuum's elastic structure. The equation suggests that the fundamental discreteness of physical processes, quantified by  $\hbar$ , is an emergent phenomenon rooted in the vacuum's ability to permit electric fields ( $\varepsilon_0$ ), and thus a built-in consequence of the elastic and geometric properties of spacetime itself.

The cubic dependence on permittivity ( $\hbar \propto \varepsilon_0^3$ ) is highly significant. It indicates that quantum action arises from a complex, volumetric self-interaction of the vacuum's compliance. It can be physically pictured as the total "elastic energy potential" that can be stored in a unit of spacetime area, a potential that is non-linearly dependent on the medium's softness.

Importantly, this result forms a perfect symmetry with the derivation for the elementary charge ( $e \propto \mu_0^3$ ). Together, they paint a complete picture:

- The **fundamental quantum of charge** ( $e$ ) is a *non-linear function of the vacuum's inertial stiffness* ( $\mu_0$ ).
- The **fundamental quantum of action** ( $\hbar$ ) is a *non-linear function of the vacuum's elastic compliance* ( $\varepsilon_0$ ).

This duality between stiffness/charge and compliance/action represents a deep, foundational symmetry of the vacuum. It suggests that the laws of electromagnetism and the laws of quantum mechanics are two sides of the same coin, both emerging from the fundamental tension between inertia and compliance in the fabric of reality.

This derivation solidifies the framework's central claim: the constants that define our physical laws are not a random assortment of numbers, but are deeply interconnected parameters that describe the single, underlying, elastic quantum vacuum.

## L. Derivation of Vacuum Constants and the Speed of Light from a Single Parameter

The ultimate test of the framework's internal consistency lies in its ability to define the fundamental constants of the vacuum,  $\mu_0$  and  $\varepsilon_0$ , and subsequently the speed of light,  $c$ , from a single, dimensionless parameter. Using the equivalence between the static and dynamic origins of the gravitational constant  $G$  (from Eq. 75) and the expression for the dynamic origin of permittivity from vacuum power principles (from Eq. 93), we can substitute Eq. (93) into Eq. (75) to obtain that:

$$\begin{aligned}\left(\frac{3}{5}\right) 4\pi \left( \mu_0^2 \cdot 2\sqrt{\frac{3}{5}4\pi} \right) &\equiv \mu_0 \alpha^2 \rightarrow \\ \mu_0 &\equiv \alpha^2 \cdot \frac{1}{16\pi \cdot \sqrt{\frac{3}{5}4\pi}} \cdot \frac{10}{3} \rightarrow \\ \boxed{\mu_0 \equiv \frac{10}{3}\alpha^3} &\end{aligned} \quad (98)$$

where we have used  $\alpha \equiv \frac{1}{16\pi \cdot \sqrt{\frac{3}{5}4\pi}}$  (87) and the derived result is a first-order approximation. Using this result, we can express  $\varepsilon_0$  purely in terms of  $\alpha$ :

$$\begin{aligned}\left(\frac{3}{5}\right) 4\pi \varepsilon_0 &\equiv \frac{10}{3}\alpha^3 \alpha^2 \rightarrow \\ \boxed{\varepsilon_0 \equiv \left(\frac{5}{3}\right)^2 \cdot \frac{1}{2\pi} \cdot \alpha^5} &\end{aligned} \quad (99)$$

Finally, with both vacuum constants defined by  $\alpha$  and geometry, we derive the speed of light using the fundamental relation  $c^2 = 1/(\mu_0\varepsilon_0)$ :

$$\begin{aligned}c^2 &\equiv \frac{1}{\left(\frac{10}{3}\alpha^3\right) \cdot \left(\frac{25}{18\pi}\alpha^5\right)} \rightarrow \\ c^2 &\equiv \frac{1}{\frac{125}{27\pi} \cdot \alpha^8} \rightarrow \boxed{c \equiv \frac{1}{\sqrt{\frac{125}{27\pi} \cdot \alpha^4}}} \end{aligned} \quad (100)$$

This result represents the culmination of the unified framework. The constants  $\mu_0$ ,  $\varepsilon_0$ , and  $c$  are no longer fundamental in their own right. They are revealed to be interdependent functions of a single, more primary parameter,  $\alpha$ . The entire framework of vacuum electrodynamics and spacetime kinematics is determined not just by a number, but by the *scalar norm* ( $\alpha$ ) of the rank-2 symmetric *structural damping tensor*,  $\zeta_{\mu\nu}$ , which encodes the vacuum's intrinsic dissipative properties.

And that is not all. The derived relationship,  $c \propto \alpha^{-4}$ , offers a profound insight into the nature of causality. The speed of light is not an

arbitrary limit but an *emergent property* dictated by the vacuum's inherent "friction". A hypothetical, perfectly frictionless vacuum ( $\alpha \rightarrow 0$ ) would permit an infinite propagation speed, rendering causality instantaneous. It is therefore the small, non-zero damping of spacetime—the inherent viscosity of the quantum oscillator lattice—that establishes a finite causal speed limit, giving the universe its structure in time.

This synthesis elevates the fine-structure constant to the role of the primary architect of physical reality. It is the scalar measure of the vacuum's fundamental dissipative geometry. The value of  $\alpha$  dictates the vacuum's inertial resistance ( $\mu_0$ ) and elastic compliance ( $\varepsilon_0$ ), and the interplay between these two properties, governed by  $\alpha$ , sets the exact value of the cosmic speed limit,  $c$ . Knowing the norm of the vacuum's damping tensor is equivalent to knowing the fundamental operational rules of spacetime.

### XIII. THE QUANTUM OF ACTION FROM SPACETIME DYNAMICS

#### A. Modal Lagrangian Density versus Effective Energy Density

A crucial distinction must be made between the *Lagrangian density* ( $\mathcal{L}$ ) used in the action integral, and the *measured vacuum energy density* ( $\rho_{\text{vac}}$ ) as an observable cosmological parameter. We propose a clear distinction between the fundamental, modal Lagrangian density ( $\mathcal{L}$ ) and the effective, measured energy density ( $\rho_{\text{vac}}$ ):

- **The Lagrangian Density** ( $\mathcal{L}$ ) describes the dynamics of a single, coherent, fundamental mode of the vacuum oscillator lattice. It is a theoretical, microscopic quantity. We identify this with the angular form:

$$\mathcal{L}_{\text{modal}} = \frac{\hbar c}{1 m^4}$$

- **The Measured Energy Density** ( $\rho_{\text{vac}}$ ) is a macroscopic, cosmological observable. It reflects the net effect of an immense number of uncoordinated vacuum oscillators, with their phases and spatial orientations being statistically random. The observable energy density is thus an effective quantity:

$$\rho_{\text{eff}} \equiv \frac{1}{2\pi} \int_0^{2\pi} \mathcal{L}_{\text{modal}} d\theta = \frac{\mathcal{L}_{\text{modal}}}{2\pi} = \frac{\hbar c}{2\pi \cdot 1 m^4}$$

Finally, note that:

- Substituting  $\lambda = 1 m$  in (61), corresponding to the characteristic scale of the unit quantum oscillator in our framework, we obtain the quantum of mass-energy for the electromagnetic field  $m = \frac{\hbar c}{1 m}$ .

- Dividing this quantum mass-energy by a volume  $V = 1 m^3$ , and considering the linear momentum  $\hbar = \frac{h}{2\pi}$  we obtain a quantum of mass density  $\rho_{\text{vac}} = \frac{\hbar c}{1 m^4}$  which corresponds with (XIII A). When transitioning from the description of a single, coherent angular mode to a macroscopic, isotropic average over all possible phases or solid angles (XIII A), we get  $\rho_{\text{eff}} \equiv \frac{\hbar c}{2\pi \cdot 1 m^4} \approx 5.03 \times 10^{-27} \text{ kg m}^{-3}$  which is in excellent agreement with cosmological measurements from the Planck satellite mission [18].

#### B. Derivation of the Quantum of Action from the Einstein-Hilbert Action

In a non-relativistic setting, an *action*  $S$  can be viewed as the time integral of the total energy (or, more precisely, the Lagrangian). In special relativity or general relativity, this idea generalizes to integrating a *Lagrangian density*  $\mathcal{L}$  over the entire spacetime volume. Formally,

$$S = \int \mathcal{L} \sqrt{-g} d^4x, \quad (101)$$

where  $g = \det(g_{\mu\nu})$  is the determinant of the metric tensor  $g_{\mu\nu}$ . The factor  $\sqrt{-g}$  ensures *general covariance* of the volume element, making the action a scalar under coordinate transformations.

The Einstein-Hilbert action [36] [37] [4] in General Relativity with a cosmological constant is typically expressed as:

$$S_{EH} = \frac{c^4}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^4x \quad (102)$$

We can equate the Einstein-Hilbert action with cosmological constant to the general equation of action we have defined as:

$$S_{EH} = \int \mathcal{L} \sqrt{-g} d^4x = \frac{c^4}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^4x \quad (103)$$

We can substitute the cosmological constant  $\Lambda$  via

$$\Lambda = 8\pi G \frac{\rho_{\text{vac}}}{c^2}, \quad (104)$$

Assuming a De Sitter universe [38] [39], one can substitute  $R = 4\Lambda$  and (104) to obtain that

$$\begin{aligned} S_{EH} &= \frac{c^4}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^4x \\ &= \frac{c^4}{16\pi G} \int 2\Lambda \sqrt{-g} d^4x \\ &= \frac{c^4}{16\pi G} \int \frac{16\pi G}{c^2} \cdot \rho_{vac} \sqrt{-g} d^4x \\ &= \int \rho_{vac} \cdot c^2 \sqrt{-g} d^4x \end{aligned} \quad (105)$$

Therefore, we identify the Lagrangian density  $\mathcal{L}$  with the energy equivalent to vacuum energy density

$$\mathcal{L} \equiv \rho_{vac} c^2 \quad (106)$$

Recent cosmological observations consistently yield an extremely small but non-zero value for  $\Lambda$ , compatible with an effective vacuum density  $\rho_{eff}$  corresponding precisely to the electromagnetic scale proposed in our unified field theory framework [40] [41]. This remarkable concordance suggests that the electromagnetic mode provides the natural interpretation of the observed cosmological vacuum state.

The fundamental geometric action must couple to the **fundamental modal energy density** of our model,  $\mathcal{L}_{\text{modal}}$ , not its statistically averaged value. Therefore, we identify the Lagrangian density as:

$$\mathcal{L} = \mathcal{L}_{\text{modal}} = \frac{\hbar c}{1 \text{ m}^4} \quad (\text{XIII A}) \quad (107)$$

Therefore, substituting, we have that

$$\rho_{vac} c^2 = \frac{\hbar c}{1 \text{ m}^4} c^2 = \frac{\hbar c^3}{1 \text{ m}^4} \quad (108)$$

In an almost flat universe, spacetime is only slightly curved, and the metric tensor  $g_{\mu\nu}$  deviates minimally from the flat Minkowski metric  $\eta_{\mu\nu}$ . Therefore, the determinant of the metric tensor  $g$  can be expressed as:

$$\sqrt{-g} \equiv 1 + \frac{1}{2} \delta g. \quad (109)$$

For practical purposes in an almost flat universe,  $\delta g$  is so small that  $\sqrt{-g} \approx 1$  is a valid approximation.

To evaluate the action for a single, fundamental "cell" of the vacuum, we must define its characteristic 4-volume ( $d^4x$ ). We construct this from the characteristic scales of the electromagnetic mode previously defined:

- The characteristic length scale:  $x_{EM} = 1 \text{ m}/c$  (IV B).
- The characteristic time scale:  $\tau_{EM} = 1 \text{ s}/c$  (VB).

The Lorentz-invariant 4-volume element is constructed from the proper time interval,  $ds = c d\tau$ , and the 3-volume. For a fundamental excitation, the time-like component of the 4-volume is  $dx^0 = c \cdot \tau_{EM} = c \cdot (1 \text{ s}/c) \equiv 1 \text{ m}$ . The spatial 3-volume is the cube of the characteristic length,  $V_3 = x_{EM}^3 = (1 \text{ m}/c)^3$ .

This defines the fundamental 4-volume of a single vacuum excitation as:

$$d^4x = (dx^0) \cdot (V_3) = (1 \text{ m}) \cdot \left(\frac{1 \text{ m}}{c}\right)^3 = \frac{(1 \text{ m})^4}{c^3}$$

As a result, substituting, one gets that

$$S_{EH} = \frac{\hbar c^3}{1 \text{ m}^4} \cdot \frac{1 \text{ m}^4}{c^3} = \hbar \quad (110)$$

### The Bridge Between the Cosmological and Quantum Realms

The result derived in Section XIII B, namely  $S_{EH} = \hbar$ , establishes a profound and direct bridge between the two pillars of modern physics: General Relativity and Quantum Mechanics. It demonstrates that the Einstein-Hilbert action—the very function that describes the large-scale dynamics of spacetime and gravity—is, under the conditions of a vacuum-dominated universe, numerically identical to the fundamental quantum of action,  $\hbar$ .

This is a fundamental statement about the nature of reality. It suggests that the "cost" of action for a fundamental 4-volume of spacetime to exist and evolve under its own vacuum energy is precisely one unit of quantum action. The macroscopic curvature described by Einstein's theory is thus shown to be built from the same fundamental, discrete "building blocks" that govern the probabilistic nature of particles at the microscopic scale. This finding strongly supports the central thesis of this work: that a single, unified quantum-oscillatory framework underlies all physical phenomena, from the cosmological to the quantum.

### C. The Quantization of Spacetime Geometry

This derivation provides a compelling physical interpretation for the quantization of gravity. By

equating the gravitational action to  $\hbar$ , we are led to the conclusion that spacetime geometry itself is quantized at its most fundamental level. The curvature of spacetime is therefore not an infinitely smooth and continuous property; rather, its manifestation in terms of action is discretized.

Within our model of the vacuum as a lattice of quantum oscillators, this result emerges as a natural consequence. A quantized lattice is expected to exhibit quantized properties. This derivation reveals that the most fundamental property of all — the action of the geometry itself — is no exception. Each "cell" of the spacetime lattice contributes exactly  $\hbar$  to the total action of the universe, reinforcing the idea that gravity is not a classical field imposed upon a quantum world, but is an emergent, large-scale manifestation of the quantum-mechanical behavior of the vacuum itself.

#### D. Self-Consistency of the Theoretical Framework

The power of this result also lies in its role as a stringent self-consistency check for the entire theoretical framework developed in this paper. The derivation began with the standard, universally accepted Einstein-Hilbert action. The only non-standard element introduced was the expression for the vacuum energy density,  $\rho_{\text{vac}}$ , which was derived independently from our quantum oscillator model and justified by its correspondence with experimental measurements.

The fact that the final output was not an arbitrary number but precisely  $\hbar$ —a cornerstone of quantum physics—is a remarkable validation of our initial postulates. It demonstrates that the principles of dimensional collapse, the interpretation of mass as an elastic response, and the modal nature of vacuum energy all work in concert within the established framework of General Relativity to produce a deeply meaningful physical result. This elevates constants like  $\hbar$  from being merely empirical inputs to being derived properties of the vacuum's geometric dynamics, providing ultimate proof of the deep interrelation between gravity and electromagnetism within the quantum vacuum, and thus closing a great conceptual loop in our understanding of the universe's fundamental constants.

## XIV. THE GEOMETRIC ORIGIN OF THE CASIMIR CONSTANT $C_c$

The magnitude of the Casimir force per unit area  $A$  between two perfectly conducting plates separated by a distance  $d$  is classically given by:

$$\frac{F_C}{A} = -\frac{\pi^2 \hbar c}{240 d^4} \approx \frac{1.3 \times 10^{-27} \text{ N} \cdot \text{m}^{-2}}{d^4},$$

where  $\hbar$  is the reduced Planck constant and  $c$  is the speed of light in a vacuum. The calculation of this expression involves handling a divergent sum using a regularization technique involving the Riemann zeta function. Our framework allows for a remarkably simple *heuristic calculation* that reproduces the correct magnitude of the force. This approach, while not formally deductive, illustrates how the effect can be understood as an intrinsic oscillatory stress within the vacuum.

Let us define the Casimir constant  $C_c$  as the quantum of "Casimir effect" force per unit of area. The zero-point energy per quantum oscillator,  $E_0 = \frac{\hbar c}{2 \cdot \lambda}$ , produces an elementary electromotive-like force  $\mathcal{E}_0 = \frac{\hbar c}{2 \cdot \lambda^2}$ , which, divided by the surface area of a sphere  $4\pi r^2$  and choosing the characteristic length scale  $\lambda = r = 1 \text{ m}$ , we obtain a first-order approximation of Casimir constant:

$$C_c = \frac{F_{qho}}{A} \approx \frac{\frac{\hbar c}{2}}{4\pi \cdot 1 \text{ m}^4} \approx 1.26 \times 10^{-27} \text{ N} \cdot \text{m}^{-2}$$

This value agrees with both theoretical estimates and experimental measurements [42, 43], and illustrates the vacuum's intrinsic capacity to sustain baseline oscillatory stress, constrained by spacetime geometry. Thus, we may write:

$$\frac{F_C}{A} = \frac{C_c}{d^4} \approx \frac{\frac{\hbar c}{2}}{4\pi \cdot (1 \text{ m})^4 \cdot d^4}, \quad (111)$$

which offers a more direct and physically intuitive way to compute the Casimir force, bypassing the need for divergent series summation or zeta-function regularization.

## XV. THE GEOMETRIC ORIGIN OF THERMODYNAMICS

### A. $k_B$ as a Quantized Force

Building on the reinterpretation of  $k_B$  as a thermo-entropic force (see Sec. V C) and the fundamental equivalence introduced earlier (Eq. 56), we now propose a novel formulation of the Boltzmann constant as an emergent relativistic force.

Assuming the dimensional equivalence  $1\text{K} \equiv 1\text{m} \equiv 1\text{s}$  within our elastic vacuum framework, we express  $k_B$  as:

$$k_B = \frac{hc}{2 \cdot 1\text{m}} \cdot \frac{1}{2\alpha \cdot 1\text{s}} = \frac{E}{a} \quad (112)$$

In this expression,  $k_B$  acquires the form of a Newtonian-like force  $F = ma$ , with the following components:

- $\frac{hc}{1\text{m}} \rightarrow m$ : The characteristic energy scale of the vacuum, associated with a fundamental quantum of mass or photon energy.
- $\frac{1}{2\alpha \cdot 1\text{s}} = \frac{\gamma}{1\text{s}} \rightarrow a$ : An effective proper acceleration, where the Lorentz-like factor  $\gamma = \frac{1}{2\alpha}$  encodes the vacuum's resistance to excitation.

Thus, the Boltzmann constant  $k_B$  emerges as a quantized force scale, representing the vacuum's intrinsic responsiveness to acceleration. In this interpretation, entropy and temperature arise from the inertial resistance of spacetime to deformation, with  $k_B$  capturing the proportionality between energetic input and induced entropic curvature.

This Unruh-inspired formulation reinforces the view that thermodynamic quantities—such as temperature, and heat—are fundamentally geometric in nature. Here,  $k_B$  bridges the gap between thermal response and relativistic motion, playing a role analogous to that of  $G$  or  $\mu_0$  in mediating the vacuum's reaction to mass or charge, respectively. In this sense,  $k_B$  can be viewed as the *thermo-entropic stiffness constant* of spacetime: a universal coupling between acceleration, information flow, and thermal excitation. This perspective helps unify quantum field theory, thermodynamics, and general relativity within a common elastic-dynamical substrate.

### B. Structural Correspondence between Thermal Radiation and Quantum Dispersion Forces

A powerful consistency check of the unified framework arises from revealing the deep connection between two seemingly unrelated phenomena: blackbody thermal radiation, governed by the Stefan-Boltzmann constant ( $\sigma$ ), and quantum intermolecular forces, governed by the London dispersion coefficient ( $C_6$ ). We will demonstrate that their dimensional correspondence is not a coincidence, but a necessary consequence of their common origin in the fluctuations of the quantum vacuum.

### The Dimensional Signature of Vacuum-Mediated Interactions

London dispersion forces are a direct manifestation of the quantum vacuum's activity. They arise from the interaction of transient dipoles induced by zero-point fluctuations of the electromagnetic field. For two neutral atoms, the interaction potential is:

$$U_{\text{London}}(r) = -\frac{C_6}{r^6}$$

where the  $C_6$  coefficient encapsulates the polarizability of the atoms and is fundamentally determined by the structure of vacuum fluctuations. In our unified dimensional framework, where energy has dimensions of length ( $[E] \equiv [L]$ ), the London coefficient carries dimensions:

$$[C_6] = [E] \cdot [L^6] \equiv [L^7].$$

Now, consider the Stefan-Boltzmann constant, whose classical expression is built from the fundamental constants of quantum mechanics, thermodynamics, and relativity:

$$\sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2}.$$

Within our dimensional system, where  $k_B$ ,  $\hbar$ , and  $c$  reduce to combinations of lengths, the Stefan-Boltzmann constant acquires:

$$[\sigma] = [\text{W m}^{-2}\text{K}^{-4}] \equiv [L^{-6}].$$

This remarkable outcome reveals that  $\sigma$  shares the inverse sixth-power length dependence characteristic of dipole-dipole vacuum interactions—a dimensional signature of shared physical origin. To formalize the connection, we define:

$$[\sigma \cdot C_6] = [L^{-6}] \times [L^7] = [L],$$

which is the dimension of energy in our framework. The product  $\sigma C_6$  thus defines an intrinsic energy scale of the vacuum, suggesting that  $\sigma$  and  $C_6$  are complementary macroscopic parameters arising from the same vacuum energy reservoir.

### Physical Interpretation: Thermal vs. Ground-State Excitations of the Vacuum

This dimensional correspondence can be physically interpreted through the modal structure of quantum field theory. Both blackbody radiation and Casimir-London forces derive from the quantization of the electromagnetic field modes within boundary conditions:

- *London and Casimir forces* emerge from the *ground-state energy* (zero-point energy) of these modes, typically expressed as  $E_0 = \frac{1}{2}\hbar\omega$ . They represent mechanical stresses exerted by the vacuum in its lowest energy configuration.
- *Stefan-Boltzmann radiation*, in contrast, arises from the *thermal excitation* of the same set of vacuum oscillators. The Stefan-Boltzmann law captures the total energy flux when these modes are thermally populated according to Planck's distribution.

Thus, London forces and blackbody radiation are not fundamentally distinct; they are two manifestations of the same quantized vacuum structure—one probing the ground state, the other the thermal state. The constants  $C_6$  and  $\sigma$  parameterize these effects at the macroscopic level, but their shared dimensionality and combined energy scaling ( $\sigma C_6$ ) highlight their unified origin. This perspective reinforces the central thesis of this work: that all physical interactions emerge as distinct modal responses of a single, elastic quantum vacuum.

*Implications for the Unified Description of Vacuum Energy and Thermodynamics*

This unification between thermal radiation and quantum dispersion forces provides a profound bridge between macroscopic thermodynamics and microscopic quantum interactions. The vacuum behaves as an elastic medium whose modal excitations, whether thermal or mechanical, dictate both the radiation laws and intermolecular forces. Accordingly, the Stefan-Boltzmann constant encapsulates not just an empirical radiation law, but a thermodynamic response of the vacuum's elastic structure. The London coefficient reflects the mechanical manifestation of the same vacuum under ground-state conditions. This duality strengthens the proposal that space-time itself possesses elastic properties, with its interaction with vacuum fluctuations giving rise to all observed forces and thermodynamic behaviors.

## Part IV: A New Fundamental Interaction: The Thermo-Entropic Field

### XVI. MOTIVATION FOR A GRAVITO-ENTROPIC FIELD

The plausibility of a structured field theory uniting gravitational and entropic dynamics is sup-

ported by a range of independent theoretical and empirical findings:

- **Gravitational wave observations**, notably those by LIGO and Virgo, confirm that the gravitational field  $\vec{g}$  can vary with time [44]. This supports the existence of dynamical couplings with an auxiliary field  $\vec{T}$ , where temporal variations in the entropic sector may induce circulation-like components in  $\vec{g}$ .
- **Black hole thermodynamics** reveals deep links between gravitational phenomena and thermodynamic quantities such as entropy and temperature [45, 46], supporting the idea that the entropic field  $\vec{T}$  is not a derivative phenomenon, but rather a fundamental component of spacetime structure.
- **Experimental confirmations of gravitomagnetic effects**, such as those from Gravity Probe B [47], show that rotating masses generate a field component dependent on mass currents. This behavior is consistent with the idea that a circulating mass flow  $\vec{J}_m$  contributes to the generation of a complementary field  $\vec{T}$ , in analogy with magnetism.
- **Thermodynamic derivations of gravitational dynamics**, such as Jacobson's approach to Einstein's equations [10] and Verlinde's emergent gravity framework [11], suggest that gravity may arise from underlying entropic principles.
- **Thermoelectric relationships** further strengthen the proposal. In condensed matter physics, temperature gradients generate electric potentials (Seebeck effect), while electric currents produce or absorb heat (Peltier effect) [48]. These two-way couplings between energy and entropy mirror the kind of mutual interactions expected in a thermo-entropic field theory. Additionally, the Unruh effect shows how temperature can emerge from acceleration, reinforcing the connection between thermodynamics and spacetime structure.

As we will see, a natural way to formalize the interplay between gravity and entropy in the fabric of spacetime is through the introduction of a *thermo-entropic field pair*  $\{\vec{g}, \vec{T}\}$ , where  $\vec{g}$  represents the gravitational field and  $\vec{T}$  denotes a thermo-entropic field, which does not denote temperature *per se*, but a circulating thermo-entropic field analogous to the magnetic field, generated by mass currents rather than charge. We coin the name *thermo-entropic* to reflect the intrinsic

duality between radial mass-induced effects (gravity) and azimuthal thermo-entropy-induced circulation (thermo-entropic fields), both encoded in the elastic response of spacetime. In the following sections, we will construct a concrete theoretical framework supporting the emergence of this thermo-entropic pair, and detailing its precise mathematical relationship with the electromagnetic pair  $\{\vec{E}, \vec{B}\}$ .

### A. The Geometrization of Thermodynamics using Hooke's Law and Fourier's Law

In line with our unified interpretation of all classical interactions as elastic responses of the vacuum medium, we can use Fourier's law of heat conduction [49] [50] and relate it to some type of induced “*thermo-entropic force*”, and Hooke's law of linear elasticity. This correspondence further supports the identification of the thermo-entropic field as a projected mode of the symmetric deformation tensor  $\mathcal{G}_{\mu\nu}$ .

We begin with the standard form of Fourier's law in vector notation:

$$\vec{q}_A = -k\nabla T, \quad (113)$$

where  $\vec{q}_A$  is the heat flux per unit area [ $\text{W m}^{-2}$ ],  $k$  is the thermal conductivity [ $\text{W m}^{-1} \text{K}^{-1}$ ], and  $\nabla T$  is the temperature gradient [ $\text{K m}^{-1}$ ]. To match the dimensional structure of other fields in the unified elastic formalism, we multiply each side by a characteristic length  $L$ , thereby defining a *line-integrated heat flux field*:

$$\vec{q} \equiv \vec{q}_A \cdot L, \quad (114)$$

which now carries units of power per unit length [ $\text{W m}^{-1}$ ], consistent with the other vector fields in our framework. Then, Fourier's law becomes:

$$\vec{q} = -k\nabla T \cdot L = -\nabla P. \quad (115)$$

where  $\nabla P$  is the power gradient [ $\text{W m}^{-1}$ ]. Here,  $P$  is interpreted as a scalar power potential whose gradient drives thermal energy flow, just as  $V$  and  $\Phi$  generate electric and gravitational fields, respectively. However, note that *in this framework, the scalar quantity  $P$  plays a role that is conceptually indistinguishable from entropy  $S$ : it quantifies the internal deformation of the vacuum associated with thermal processes*. Indeed, since both  $P$  and  $S$  are dimensionally equivalent in the collapsed vacuum-elastic formalism, and since they both act as sources of thermodynamic flow when modulated by temperature, *we can regard entropy as the natural scalar field driving thermo-entropic deformation*.

Thus, from the standpoint of vacuum elasticity, entropy becomes a geometrically grounded scalar field, whose gradients yield observable thermal forces and fluxes. The identification of  $S$  with the scalar potential of the thermo-entropic field completes its structural analogy with gravitation and electromagnetism.

In exact parallel with the voltage  $\mathcal{E} = \int \vec{E} \cdot d\vec{\ell}$  of electrostatics, the thermo-entropic field defines a *thermo-entropic electromotive force*

$$\mathcal{E}_T \equiv \int_{\ell} \vec{q} \cdot d\vec{\ell} \quad (116)$$

with units of power [W]. In the dimension-collapsed vacuum-elastic system adopted here, powers are dimensionless, so  $\mathcal{E}_T$  plays exactly the same algebraic role as an (dimensionless) electric EMF.

Given the dimensional equivalence between power and entropy in the vacuum-elastic unit system, the quantity  $\mathcal{E}_T$  can also be interpreted as a net entropy difference between two thermal regions. Thus, the thermo-motive force becomes not only a measure of energy flux, but also a geometric manifestation of entropic imbalance in the vacuum lattice—driving deformation analogously to how electric potential drives charge.

As a result, each classical field admits a corresponding integral expression that encodes its action over an extended region. The following table summarizes these correspondences:

Phenomenon	Field	Expression	Physical meaning
Heat	$\vec{q}$	$\int \vec{q} \cdot d\vec{A}$	Thermal flux
Electricity	$\vec{E}$	$\int \vec{E} \cdot d\vec{A}$	Electric flux
Magnetism	$\vec{B}$	$\int \vec{B} \cdot d\vec{A}$	Magnetic flux
Gravity	$\vec{g}$	$\int \vec{g} \cdot d\vec{A}$	Gravitational flux

If we denote by  $\Delta T$  the temperature differential between adjacent isothermal layers, we can express the thermo-entropic response in a strictly Hookean form:

$$S = \kappa_{\text{dyn}} \Delta T \quad (117)$$

Here,  $S$  represents the entropy associated with the deformation of the vacuum medium, and  $\kappa_{\text{dyn}}$  acts as a dynamic entropic stiffness with dimensions [ $L^{-1}$ ] in the vacuum-elastic unit system. This formulation reinforces the interpretation of entropy as a scalar elastic displacement field, and  $\Delta T$  as its driving cause.

Hookean elasticity	Thermo-entropic
$F$ (mechanical force)	$P$ (power) / $S$ (entropy)
$\Delta x$ (displacement)	$\Delta T$ (Temperature diff.)
$k$ (spring constant, $[L^{-1}]$ )	$\kappa_{\text{dyn}}$ ( $[L^{-1}]$ or $\text{W s}^{-1}$ )

Therefore, the thermo-entropic field integrates consistently into our unified tensorial framework as a Hookean deformation mode, whose force-like response is power and whose generalized displacement is temperature difference.

### B. Summary of structural analogies between elastic responses of the vacuum

We present below the structural correspondence between the linearized Fourier's law, Gauss's law in electrostatics, and Newton's law of gravitation:

Concept	Fourier's L.	Gauss's L.	Newton's L.
Source	$\nabla \cdot \vec{q}$	$\nabla \cdot \vec{E}$	$\nabla \cdot \vec{g}$
Potential	$P \equiv S$	$V$	$\Phi$
Field	$\vec{q} = -\nabla S$	$\vec{E} = -\nabla V$	$\vec{g} = -\nabla \Phi$
Poisson eq.	$\nabla^2 S = k_B \rho_{\text{temp}}$	$\nabla^2 V = -\frac{\rho_e}{\epsilon_0}$	$\nabla^2 \Phi = 4\pi G \rho_m$

where  $\rho_{\text{temp}}$  represents a localized temperature density or distribution acting as the source of entropic deformation. Each of these field laws describes how a scalar potential gives rise to a vector field through a gradient operation, and how the divergence of that field connects to a source density via a Poisson-type equation. In this structural analogy:

Quantity	Electromagnetism	Gravito-entropism
Source	Electric charge ( $q$ )	Mass ( $m$ )
Main field (circulatory)	$\vec{B} = \frac{\mu_0 \vec{I}}{2\pi r} \hat{\theta}$	$\vec{\mathcal{T}} = \frac{k_B \vec{I}_m}{2\pi r} \hat{\theta}$
Derived field (radial)	$\vec{E} = \frac{K_e Q}{r^2} \hat{r}$	$\vec{g} = \frac{GM}{r^2} \hat{r}$
Coupling constant	$\mu_0$	$k_B = \mu_0/c^2$
Gauss's law	$\nabla \cdot \mathbf{E} = 4\pi K_e \rho_q$	$\nabla \cdot \mathbf{g} = -4\pi G \rho_m$
No-monopole law	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathcal{T} = 0$
Faraday's law	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{g} = -\frac{\partial \mathcal{T}}{\partial t}$
Ampère-Maxwell law	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\nabla \times \mathcal{T} = k_B \mathbf{J}_m + k_B \epsilon_0 \frac{\partial \mathbf{g}}{\partial t}$

TABLE I. Comparison of Maxwell-like laws for electromagnetism and the proposed gravito-entropic sector.

Here,  $\vec{I}_m$  and  $\rho_m$  respectively represent mass current and mass density. The product  $k_B \epsilon_0 = \frac{1}{c^4}$  plays a central role in the dynamic coupling of the

- The entropy  $S$  plays the role of a scalar thermal potential,
- The linearized heat flux  $\vec{q}$  is analogous to the electric field  $\vec{E}$  or the gravitational field  $\vec{g}$ ,
- And the Laplacian  $\nabla^2 S$  captures thermo-entropic curvature, in full parallel with electrostatic and gravitational curvature.

The thermo-entropic field, driven by entropy gradients and governed by thermal curvature, thereby integrates as a scalar deformation mode within the elastic manifold defined by the symmetric tensor  $\mathcal{G}_{\mu\nu}$  (XVIII B). This geometrization of heat completes the triad of scalar sources—mass, charge, and temperature—unifying their corresponding field interactions as elastic responses of the vacuum encoded in the symmetric tensor  $\mathcal{G}_{\mu\nu}$ .

### XVII. THE GRAVITO-MAXWELL EQUATIONS

Based on the similarities between the electromagnetic pair  $\{\vec{E}, \vec{B}\}$  and the proposed thermo-entropic pair  $\{\vec{g}, \vec{\mathcal{T}}\}$ , we propose a system of field equations mirroring Maxwell's equations. The gravitational field plays the role of the electric field, while the entropic field assumes the role of the magnetic field.

Below can be found a table summarizing the main derivations and relationships:

of the entropic field  $\vec{T}$ , and encodes the universal stiffness of the vacuum to longitudinal, thermo-entropic excitations. Remarkably, this product can also be written as  $k_B \varepsilon_0 \equiv \hbar/m^2$ , suggesting that the minimal entropic deformation of the vacuum carries a quantized action density. This reinforces the interpretation of  $k_B \varepsilon_0$  as a modal invariant of the elastic vacuum: a fundamental coupling constant linking action, entropy, and geometry in the thermo-entropic sector.

### XVIII. CONSEQUENCES AND VALIDATION OF THE THERMO-ENTROPIC FIELD

#### A. Fundamental parameters of the thermo-entropic field

From our proposal, we can derive the fundamental parameters of the thermo-entropic field just dividing the parameters obtained for the electromagnetic field by  $c^2$ :

- We obtain the quantum of mass-energy for the thermo-entropic field  $m_{entr} = \frac{\hbar}{2\pi c \cdot 1} m \approx 5.6 \times 10^{-44}$  kg.
- We obtain a quantum of mass density  $\rho_{entr} = \frac{\hbar}{2\pi c \cdot 1} m^4$  kg m<sup>-3</sup>.
- We can set the action as  $\frac{\hbar}{c^2}$ , which matches the result derived previously (V C).

#### B. Deriving Unruh effect from the thermo-entropic field parameters and Newton's law

There are several checks that we can perform to further justify the validity of the fundamental parameters derived for the thermo-entropic field. In this subsection, we will focus on showing that the Unruh effect can be directly derived from the application of the fundamental expression of mass-energy for the thermo-entropic field and Newton's Law.

The **Unruh effect** [51] states that an observer with constant proper acceleration  $a$  in vacuum perceives a thermal bath at a temperature

$$T_{\text{Unruh}} = \frac{\hbar a}{2\pi c k_B}. \quad (118)$$

Rearranging gives

$$k_B = \frac{\hbar a}{2\pi c T_{\text{Unruh}}}. \quad (119)$$

We have already shown how  $k_B$  can be treated dimensionally as a force (V C). Also, we have postulated that the expression of mass-energy for the

thermo-entropic field is  $m_{entr} = \frac{\hbar}{2\pi c \cdot \lambda}$ . Using the equivalence  $[L] \equiv [T_{emp}]$ , we can easily see that the equation of the Unruh effect can be rewritten as

$$k_B = m_{entr} \cdot a \quad (120)$$

Showing how Unruh's effect can be derived from the application of Newton's law to the derived parameters of the thermo-entropic field.

## Part V: A Unified Field Theory from a Symmetric Tensor

### XIX. CONCEPTUAL HIERARCHY OF THE UNIFIED FIELD FORMULATION

Below we show the conceptual hierarchy of the unified field formulation that we will detail briefly throughout the next sections:

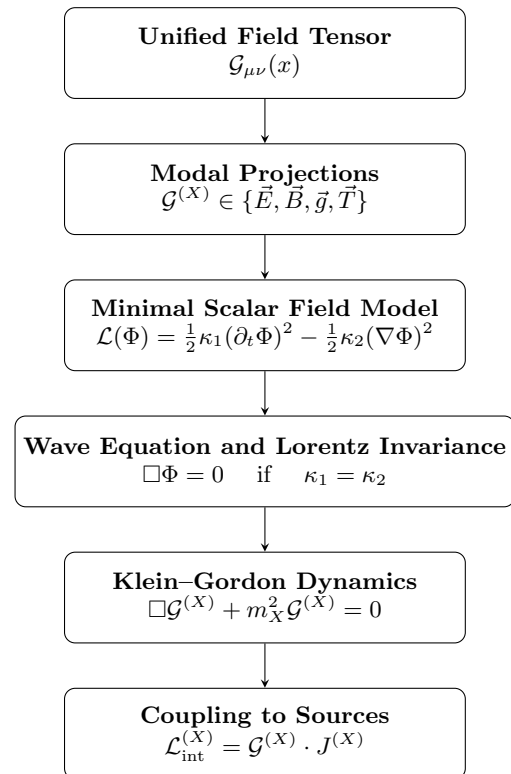


FIG. 1. Conceptual hierarchy of the unified field formulation: from geometric tensor structure to modal dynamics.

## XX. THE UNIFIED FIELD EQUATION AND THE UNIVERSAL POTENTIAL

### A. The Geometro-Elastic Principle and the Laplacian Operator

The central thesis of this work is that the vacuum is an elastic medium, and its state of deformation is described by a single, symmetric tensor,  $\mathcal{G}_{\mu\nu}(x)$ . To describe the static configuration of this field around a source, we must identify the correct mathematical operator that governs the equilibrium of an elastic continuum.

From first principles of physics and engineering, the state of a continuous medium in equilibrium (like a stretched membrane or a stressed solid) is described by the **Laplacian operator**,  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ . The Laplacian of a field at a point represents the difference between the field's value at that point and the average value in its immediate vicinity. Therefore:

- A region where  $\nabla^2 \mathcal{G}_{\mu\nu} = 0$  (Laplace's equation) is in **elastic equilibrium**, with no net internal stress.
- A region where  $\nabla^2 \mathcal{G}_{\mu\nu} = -J_{\mu\nu}$  (Poisson's equation) contains a **source of stress**,  $J_{\mu\nu}$ .

Given our geometro-elastic framework, the 3D Laplacian is thus the most natural and correct operator to describe the static deformation of the vacuum in response to a source.

### B. The Universal Potential as the Green's Function

The fundamental response of this elastic vacuum to a localized, point-like source ( $J_{\mu\nu}$  represented by a Dirac delta function) is given by the solution to the Poisson equation. This solution is the **Green's function** for the Laplacian in three dimensions:

$$\text{Solution for a point source} \propto \frac{1}{4\pi r} \quad (121)$$

This establishes the  $1/r$  potential as the **universal, fundamental response** of the 3D elastic vacuum to any localized excitation. It is not specific to any one force, but is the underlying mathematical structure for all static interactions.

#### *Eigenstates of the Discrete Laplacian and Identification of Physical Modes*

In a three-dimensional lattice, the Laplacian operator can be discretized, and its eigenstates (or

proper modes)  $\phi_n(\mathbf{r})$  satisfy the eigenvalue equation:

$$\nabla_{\text{disc}}^2 \phi_n(\mathbf{r}) = -\lambda_n \phi_n(\mathbf{r}),$$

where  $\lambda_n$  denotes the spectrum of eigenvalues. We propose that physical field modes are not arbitrary deformations, but correspond to specific linear combinations of these fundamental eigenstates:

- **Radial Modes** ( $\vec{E}, \vec{g}$ ): These fields, characterized by spherical symmetry, correspond to Laplacian eigenstates with radial symmetry. In the continuum limit, they naturally recover the  $1/r$  profile.
- **Azimuthal Modes** ( $\vec{B}, \vec{T}$ ): These fields represent torsional or circulatory deformations and correspond to eigenstates with rotational (vortical) symmetry. The degeneracy in the eigenvalue spectrum for these modes may be responsible for the emergence of local gauge symmetries, providing a rationale for how a symmetric fundamental tensor  $\mathcal{G}_{\mu\nu}$  can give rise to derived antisymmetric force fields, as discussed in Section XXII B.

### C. Validation: The Hierarchy of Physical Fields

Within the proposed unified framework, fundamental fields derived from distinct physical interactions exhibit a similar structure modulated by dimensionless factors derived from the theory. Specifically, the electric field ( $\vec{E}$ ), magnetic field ( $\vec{B}$ ), gravitational field ( $\vec{g}$ ), and the thermo-entropic field ( $\vec{T}$ ) (XVI) arise based on the derivations within this framework, and take the form:

$$\begin{aligned} (i) \quad \vec{E} &= \frac{e}{4\pi\epsilon_0 r^2} \hat{r} \equiv \frac{2\alpha \cdot \mu_0 c^2}{c^2 \cdot 4\pi \cdot r} \hat{r} = 2\alpha \cdot \frac{\mu_0}{4\pi r} \hat{r} \\ (ii) \quad \vec{B} &= \frac{\mu_0 I}{4\pi r} \hat{\theta} = \frac{\mu_0 \cdot c}{4\pi \cdot r} \hat{\theta} = c \cdot \frac{\mu_0}{4\pi r} \hat{\theta} \equiv \frac{\vec{E} \cdot c}{2\alpha} \\ (iii) \quad \vec{g} &= \frac{Gm}{r^2} \hat{r} \equiv \frac{\mu_0 \alpha^2 \cdot \hbar c}{r^2 \cdot 2 \cdot m} \hat{r} \equiv \alpha^2 \cdot \frac{\hbar c}{1 \cdot m^2} \cdot \frac{\mu_0}{4\pi r} \hat{r} \\ (iv) \quad \vec{T} &= \frac{k_B \cdot 2\alpha \cdot I}{4\pi r} \hat{\theta} = \frac{2\alpha}{c^2} \cdot \frac{\mu_0}{4\pi r} \hat{\theta} \equiv \frac{\vec{g} \cdot c}{2\alpha} \end{aligned} \quad (122)$$

where we have used  $e = \frac{2\alpha \cdot 1 \cdot C}{c^2}$ ,  $m = \frac{\hbar c}{2 \cdot 1 \cdot m}$  (XIII A),  $I = c$  (electromagnetic mode) and  $I = \frac{1}{c}$  (thermo-entropic mode), together with the equivalences of (9).

Note that all the static fields adopt the generic form

$$\vec{\Phi}_X(r) = \frac{\mu_0}{4\pi r} \cdot C_X \cdot \hat{e}_X,$$

where  $X \in \{E, B, g, T\}$  and  $C_X$  is a dimensionless coefficient. The direction  $\hat{e}_X$  indicates the field's spatial orientation — either radial or azimuthal.

In this formulation, the characteristic expressions for the fundamental fields are constructed from the elementary contribution of discrete quantum oscillators. The magnetic field, for instance, is not derived from the standard expression for an infinite wire ( $\vec{B} = \mu_0 I / 2\pi r$ ), but rather from the Biot–Savart law applied to a localized oscillatory mode, yielding the expression  $\vec{B} = \mu_0 I / 4\pi r$ , which better reflects the point-like, modular nature of the vacuum excitations in this framework. The mass  $m$  has been taken from the zero-point energy associated with a confined quantum harmonic oscillator mode of the vacuum, becoming an emergent quantity from the fundamental vacuum oscillation modes. This aligns with the conception of mass as a localized deformation in some elastic medium, and the unification of field modes under a shared geometric and dynamical substrate.

Note that, while the universal structural form of the fields emerges naturally from the Laplacian formalism, their relative magnitudes, encapsulated in the coefficients  $C_X$ , are a direct consequence of the equivalence principles postulated in Part I. All four field expressions share a universal structural factor:

$$\frac{\mu_0}{4\pi r}$$

which can be interpreted as the universal elastic modulus of the vacuum. This quantity encapsulates the intrinsic coupling between elastic excitations and physical sources such as charge, mass, and temperature. Notably, this coupling is not arbitrary: using our fundamental identity (56)

$$\mu_0 \cdot e \equiv \frac{h \cdot c}{1 \cdot m},$$

one can easily derive

$$\frac{\mu_0}{4\pi} = \frac{\hbar c}{2 \cdot 1 \cdot m \cdot e} = \frac{E_0}{e},$$

where  $E_0 = \hbar c / (2 \cdot 1 \cdot m)$  is identified as the zero-point energy of a fundamental oscillator mode of the vacuum, and simultaneously interpreted as the rest mass energy associated with a unit deformation cell. This expression encodes  $\mu_0 / 4\pi$  as some *fundamental vacuum deformation*, intrinsic to all field excitations and independent of the specific interaction, akin to some *deformation current*: the amount of vacuum energy deformation (in the form of rest mass) per unit of charge. In this sense, it quantifies the vacuum's elastic response rate to localized sources—be they of electric or gravitational nature.

#### D. Eigenvalues as Geometric Origin of Physical Constants

The identification of the physical field modes with eigenstates of the discrete Laplacian (XXB) allows us to interpret the eigenvalues  $\lambda_n$  as geometric stiffness coefficients of the vacuum lattice. Importantly, these eigenvalues are not arbitrary: their degeneracies and ratios encode internal symmetries among field sectors.

In particular, the observed structure of the unified fields reveals a remarkable hierarchy: each *azimuthal* field mode ( $\vec{B}, \vec{T}$ ) emerges as a rescaled counterpart of its corresponding *radial* field mode ( $\vec{E}, \vec{g}$ ) through a universal scaling factor:

$$\begin{aligned} \vec{B} \cdot \frac{2\alpha}{c} = \vec{E} &\rightarrow \vec{B} \cdot I_{max} = \vec{E} \\ \vec{T} \cdot \frac{2\alpha}{c} = \vec{g} &\rightarrow \vec{T} \cdot I_{max} = \vec{g}, \end{aligned} \quad (123)$$

where  $I_{max} \equiv G_{Glob}$  is the fundamental current we derived previously (69). This reveals a profound symmetry in the structure of the fundamental interactions, and suggests an inversion of the traditional perspective: a radial force field (like  $\vec{E}$  or  $\vec{g}$ ) can be interpreted as emerging from a primary circulatory field ( $\vec{B}$  or  $\vec{T}$ ) when it is amplified by the vacuum's maximum current amplitude,  $I_{max}$ . Conventionally, static potential fields (like  $\vec{E}$ ) are considered primary, from which circulatory fields (like  $\vec{B}$ ) are derived via dynamics and source currents. Our framework, however, allows for a compelling alternative interpretation: the circulatory or torsional modes of the vacuum ( $\vec{B}, \vec{T}$ ) may be the more fundamental excitations. In this view, the radial force fields ( $\vec{E}, \vec{g}$ ) emerge as secondary, effective phenomena when the primary circulatory fields are amplified by the vacuum's intrinsic current capacity,  $I_{max}$ .

*A Hydrodynamic Analogy.* To provide a more intuitive physical picture for the relationship  $\vec{E} \equiv \vec{B} \cdot I_{max}$ , we can draw a direct analogy with continuum hydrodynamics. If we model the vacuum as an idealized, incompressible fluid, then the azimuthal fields,  $\vec{B}$  and  $\vec{T}$ , are analogous to a localized vorticity or a circulatory flow field within this fluid. According to fluid dynamics, such a circulatory motion generates a pressure gradient directed towards the center of the vortex. This pressure gradient, in turn, gives rise to a net radial force field. This emergent radial force is the analogue of the fields  $\vec{E}$  and  $\vec{g}$ .

In this picture, the constant  $I_{max}$  plays the role of a fundamental constitutive property of the vacuum fluid. It determines the efficiency of this mode conversion, quantifying how much

radial force is generated for a given amount of circulation. A higher  $I_{\max}$  would correspond to a medium in which vorticity more effectively induces a pressure gradient, thus generating a stronger radial force field. Therefore,  $I_{\max}$  is not merely a current, but a fundamental conversion factor of the vacuum medium itself. It acts as the *universal coupling constant* that connects the azimuthal (circulatory) and radial (potential) modes of the unified field tensor  $\mathcal{G}_{\mu\nu}$ .

The fact that the *same* constant,  $I_{\max}$ , governs the transformation for both the electromagnetic pair  $(\vec{E}, \vec{B})$  and the gravito-entropic pair  $(\vec{g}, \vec{T})$  is a powerful statement of unification. It implies that the underlying mechanism connecting circulatory dynamics to static forces is a universal property of spacetime's elastic structure, independent of the specific nature of the interaction.

In summary, the hierarchy of physical fields observed in the unified theory—radial versus azimuthal, electromagnetic versus thermo-entropic—is rooted in the spectral geometry of the discrete Laplacian on the vacuum lattice. Each field is a modal projection of the elastic deformation tensor  $\mathcal{G}_{\mu\nu}$ , characterized by its eigenfunction class, coupling coefficient, and orientation. These relations consolidate the geometric unity of the theory, and establish the Laplacian as the fundamental generator of the spacetime excitation spectrum.

## XXI. MINIMAL SCALAR FIELD MODEL FOR MODAL EXCITATIONS

To provide a dynamical description of the modal excitations of the unified tensor  $\mathcal{G}_{\mu\nu}$ , we can construct a minimal model that captures their essential propagation features. This model serves as an effective description for a single projected mode, which we can denote as a scalar field  $\Phi(x)$ . The dynamics of this effective field are not arbitrary, but are a direct consequence of the general Lagrangian for the unified field introduced in Section II and fully detailed in Section XXII A.

Recall that the kinetic part of the unified field Lagrangian is governed by a single, fundamental stiffness constant  $\kappa$ :

$$\mathcal{L}_{\text{kinetic}} = \frac{1}{2} \kappa (\partial^\alpha \mathcal{G}^{\mu\nu}) (\partial_\alpha \mathcal{G}_{\mu\nu})$$

We can project these general dynamics onto a single scalar mode  $\Phi(x)$ , which will naturally contain terms for the kinetic and potential energy, analogous to a harmonic oscillator, and the two couplings (or "rigidities") usual for harmonic oscilla-

tory systems,  $\kappa_1$  and  $\kappa_2$ , which are effective combinations of physical constants (e.g.,  $\varepsilon_0$ ,  $\mu_0$ ,  $G$ ,  $k_B$ , etc.) under different substitutions depending on the mode of the elastic-oscillatory manifestation of the common field. Then, one obtains the following *minimal* Lagrangian density that we already used in 35:

$$\mathcal{L}(\Phi) = \frac{1}{2} \kappa_1 (\partial_t \Phi)^2 - \frac{1}{2} \kappa_2 (\nabla \Phi)^2 \quad (124)$$

Here,

- $\kappa_1$  controls the "inertial" or kinetic response of the field mode,
- $\kappa_2$  represents the elastic/spatial rigidity of the field mode.

The corresponding *action*  $S$  is given by the integral

$$S[\Phi] = \int d^4x \mathcal{L}(\Phi, \partial\Phi; \kappa_1, \kappa_2) \quad (125)$$

Applying the principle of least action,  $\delta S = 0$ , yields the Euler–Lagrange equation:

$$\kappa_1 \frac{\partial^2 \Phi}{\partial t^2} - \kappa_2 \nabla^2 \Phi = 0. \quad (126)$$

This *single* partial differential equation governs the field  $\Phi$ . Depending on how we identify  $\kappa_i$  with physical constants and  $\Phi$  with different spacetime deformations (e.g., mass, charge, temperature), we recover the different field modes.

### A. Relativistic Compatibility and Spacetime Formalism

To ensure compatibility with special relativity, we introduce a four-dimensional spacetime coordinate:

$$X^\mu = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}, \quad \text{with metric } \eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1). \quad (127)$$

We define the spacetime derivatives:

$$\partial_\mu = \frac{\partial}{\partial X^\mu} = \begin{pmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \quad (128)$$

The standard Lorentz-invariant kinetic structure is encoded in the d'Alembertian operator:

$$\square \Phi \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu \Phi = -\frac{\partial^2 \Phi}{\partial t^2} + \nabla^2 \Phi. \quad (129)$$

The equation of motion from our minimal field Lagrangian reads:

$$\kappa_1 \frac{\partial^2 \Phi}{\partial t^2} - \kappa_2 \nabla^2 \Phi = 0. \quad (130)$$

To write this in a Lorentz-invariant form proportional to  $\square \Phi = 0$ , we compare:

$$\kappa_1 \frac{\partial^2 \Phi}{\partial t^2} = \kappa_2 \nabla^2 \Phi \implies \frac{\partial^2 \Phi}{\partial t^2} = \frac{\kappa_2}{\kappa_1} \nabla^2 \Phi. \quad (131)$$

Rewriting the d'Alembertian as:

$$\square \Phi = -\frac{\partial^2 \Phi}{\partial t^2} + \nabla^2 \Phi = 0,$$

we see that Lorentz invariance requires:

$$\frac{\kappa_2}{\kappa_1} = 1 \implies \kappa_1 = \kappa_2. \quad (132)$$

Therefore, the field theory is manifestly Lorentz-invariant if and only if the rigidity constants match:  $\kappa_1 = \kappa_2$ . This is a profound physical statement: it reflects that in a Lorentz-covariant theory built upon a single field, the inertial and elastic responses to variations must originate from the same isotropic, fundamental stiffness  $\kappa$  of the vacuum substrate. This ensures the action transforms as a scalar and the field equation becomes the standard wave equation:

$$\square \Phi = 0. \quad (133)$$

Alternatively, one may directly enforce Lorentz invariance in the Lagrangian density by writing:

$$\mathcal{L}_{\text{Lorentz}}(\Phi) = \frac{1}{2} \kappa \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi, \quad (134)$$

where  $\kappa$  encodes the rigidity of the mode and can later be matched to physical constants depending on the interpretation. In this more general framework, if one allows  $\kappa_1 \neq \kappa_2$ , the resulting dynamics describe propagation in a medium with anisotropic or symmetry-breaking features, which may be interpreted as emergent properties or effective behaviors in specific physical regimes (e.g., entropy-driven diffusion or gravitational strain), allowing the unified field to encompass richer phenomena under the same fundamental structure.

This scalar field model, though minimal, captures the essential dynamics of vacuum excitations. It shows how each modal projection of the unified field tensor  $\mathcal{G}_{\mu\nu}$  can be described in terms of a wave-like field obeying Lorentz-invariant dynamics. This prepares the ground for a more detailed relativistic formulation in terms of Klein–Gordon dynamics and projection structures consistent with general relativity. In the following section, we generalize this framework to include mass terms, explicit coupling to sources, and consistent tensorial projections compatible with general relativity.

## XXII. THEORETICAL FOUNDATIONS: KLEIN-GORDON DYNAMICS AND GENERAL RELATIVITY CONSISTENCY

### A. Lagrangian framework for modal excitations

Building on the minimal scalar field formulation—which established that any modal projection must obey Lorentz-invariant dynamics—we now construct a canonical framework for these excitations. This step transitions from the fundamental Lagrangian governed by the stiffness constant  $\kappa$  to an effective, canonical Lagrangian for each mode. This is achieved via field renormalization, a standard procedure in field theory where the physical constant  $\kappa$  is absorbed into the field's definition (e.g.,  $\mathcal{G}' = \sqrt{\kappa} \mathcal{G}$ ) to simplify the kinetic term.

This formulation allows the modal excitations of the unified tensor  $\mathcal{G}_{\mu\nu}$  to be described in terms of renormalized, scalar-like fields  $\mathcal{G}^{(X)}$ . The dynamics of these modes, including potential mass terms that arise from the full theory's potential term  $V(\mathcal{G}_{\mu\nu})$ , can be described by the following canonical Klein–Gordon Lagrangian:

$$\mathcal{L}_X = \frac{1}{2} \partial^\mu \mathcal{G}^{(X)} \partial_\mu \mathcal{G}^{(X)} - \frac{1}{2} m_X^2 \left( \mathcal{G}^{(X)} \right)^2, \quad (135)$$

where  $\mathcal{G}^{(X)}$  represents a renormalized projection of the full tensor  $\mathcal{G}_{\mu\nu}$  along mode  $X \in \{E, B, g, T\}$ . The term  $m_X$  is not a new fundamental parameter, but an *effective mass* that a mode acquires due to the self-interaction potential of the underlying unified field. This Klein–Gordon-type Lagrangian provides a Lorentz-invariant basis for describing both massless ( $m_X = 0$ ) and massive field modes within the unified elastic framework.

The Euler–Lagrange equation yields:

$$\square \mathcal{G}^{(X)} + m_X^2 \mathcal{G}^{(X)} = 0, \quad (136)$$

which reduces to the Klein–Gordon equation for free scalar propagation in Minkowski space.

Although the Lagrangian  $\mathcal{L}_X$  describes free fields, coupling to physical sources can be incorporated via minimal interaction terms of the form:

$$\mathcal{L}_{\text{int}}^{(X)} = \mathcal{G}^{(X)}(x) \cdot J^{(X)}(x), \quad (137)$$

where  $J^{(X)}$  is an effective source density corresponding to charge, mass, entropy flux, etc. These terms play an analogous role to the coupling  $A_\mu J^\mu$  in electrodynamics. Since  $\mathcal{G}^{(X)}$  represents a modal projection of the unified tensor  $\mathcal{G}_{\mu\nu}$ , the

coupling is assumed to act only on the relevant scalarized or vectorial component associated with the physical mode.

A more general coupling scheme could link the full tensor to the energy–momentum content of matter via:

$$\mathcal{L}_{\text{int}} = \mathcal{G}_{\mu\nu}(x) T^{\mu\nu}(x), \quad (138)$$

from which each modal interaction  $\mathcal{G}^{(X)} J^{(X)}$  would arise as a projection or contraction. This formulation ensures full compatibility with general relativistic coupling schemes.

## B. Static solutions

In the static limit and for massless modes ( $m_X = 0$ ), the equation (137) reduces to:

$$\nabla^2 \mathcal{G}^{(X)}(\vec{r}) = -J^{(X)}(\vec{r}), \quad (139)$$

with  $J^{(X)}$  being an effective source term. For a point-like unit source located at the origin,  $J^{(X)}(\vec{r}) = \delta^{(3)}(\vec{r})$ , the Green's function solution is:

$$\mathcal{G}^{(X)}(r) = \frac{1}{4\pi r}. \quad (140)$$

Multiplying this fundamental response by a dimensionless coupling  $C_X$  and by the universal deformation factor  $\mu_0$  yields the physical field expression:

$$\boxed{\vec{\Phi}_X(r) = \mu_0 \cdot \frac{C_X}{4\pi r} \cdot \hat{e}_X} \quad (141)$$

which matches the expression derived in section XX C.

Each mode  $\mathcal{G}^{(X)}$  can be formally extracted from the full tensor using projection operators  $P_{(X)}^{\mu\nu}$  acting on  $\mathcal{G}_{\mu\nu}$ :

$$\mathcal{G}^{(X)}(x) := P_{(X)}^{\mu\nu} \mathcal{G}_{\mu\nu}(x), \quad (142)$$

ensuring that the decomposition is orthogonal, complete, and compatible with spacetime symmetries.

### *Projected Field Modes and Geometric Interpretation of $\mathcal{G}_{\mu\nu}$*

We propose that the field  $\mathcal{G}_{\mu\nu}$  is symmetric and real, with ten independent components:

- $\mathcal{G}_{00}$  encodes scalar deformations (electrostatic, gravitational).

- $\mathcal{G}_{0i}$  encodes torsional modes (magnetic field analog).
- $\mathcal{G}_{ij}$  represents spatial-shear or volume modes (gravitational and thermo-entropic analogs).

Their geometric meaning is determined by symmetry (e.g., spherical) and energy scale. Together, these components describe the elastic response of the vacuum to localized excitation. Each scalar field  $\mathcal{G}^{(X)}$  is identified as a projection of a specific component or contraction of the unified tensor  $\mathcal{G}_{\mu\nu}$ , depending on symmetry and field geometry. Examples include:

$$\begin{aligned} \mathcal{G}^{(E)} &:= \mathcal{G}_{00}, \\ \mathcal{G}^{(B)} &:= \sqrt{\mathcal{G}_{0i} \mathcal{G}^{0i}}, \\ \mathcal{G}^{(g)} &:= \text{Tr}(\mathcal{G}_{ij}), \\ \mathcal{G}^{(T)} &:= \sqrt{\mathcal{G}_{ij} \mathcal{G}^{ij}}. \end{aligned}$$

These choices reflect radial vs. azimuthal structure and are consistent with the interpretation of the fields as spatially oriented deformation modes. Although  $\mathcal{G}^{(B)}$  and  $\mathcal{G}^{(T)}$  are scalar in dynamics, the physical fields  $\vec{B}$  and  $\vec{T}$  acquire their azimuthal direction  $\hat{\theta}$  through the geometric structure of the excitation mode (e.g., torsional boundary condition). This is analogous to standing wave patterns in elastic continua where mode functions are scalar but correspond to vectorial deformations.

A rigorous modal decomposition would require defining a complete set of orthogonal projection operators  $P_{(X)}^{\mu\nu}$  that extract each physical mode from  $\mathcal{G}_{\mu\nu}$ . In this initial formulation, we adopt symmetry-guided identifications consistent with the observed field patterns, while leaving the development of a full operatorial decomposition to future refinement.

### *Emergence of Antisymmetric Excitations*

Although the unified field  $\mathcal{G}_{\mu\nu}$  is symmetric, antisymmetric field excitations—such as those encoded in the electromagnetic field tensor  $F_{\mu\nu}$ —can emerge as derived objects from its spacetime derivatives. Specifically, we define generalized field strengths:

$$F_{\mu\nu}^{(X)} := \partial_\mu \mathcal{G}_{\nu\rho}^{(X)} - \partial_\nu \mathcal{G}_{\mu\rho}^{(X)}, \quad (143)$$

where  $\rho$  is a fixed index associated with the geometric direction of deformation—typically radial ( $\rho = 0$ ) for electric or gravitational modes, and azimuthal ( $\rho = i$ ) for magnetic or entropic torsional analogs.

This construction reflects the idea that local antisymmetric excitations, such as rotational or torsional responses, arise as gradients or curls of symmetric deformations in an elastic continuum. In particular:

- For  $X = E$ , choosing  $\rho = 0$  yields a generalized electric-type field strength.
- For  $X = B$ , choosing  $\rho = i$  (azimuthal direction) produces magnetic-like curls.
- For  $X = T$  and  $X = g$ , similar constructions could yield thermo-entropic and gravitodynamic vorticities, respectively.

This formalism embeds antisymmetric field behavior directly into the derivative structure of the symmetric tensor  $\mathcal{G}_{\mu\nu}$ . It suggests that classical field strengths  $F_{\mu\nu}$ , typically postulated as fundamental, may instead arise as secondary geometric quantities, traces of deeper symmetric modes of the quantum-elastic vacuum.

Furthermore, this opens a natural path to defining the field dynamics from an action principle involving scalar invariants of the form:

$$\mathcal{L}_X \sim F_{\mu\nu}^{(X)} F^{(X)\mu\nu},$$

which parallels the standard electromagnetic Lagrangian, but grounds it in the unified tensorial origin of  $\mathcal{G}_{\mu\nu}$ .

A rigorous generalization would involve introducing projection operators  $P_{(X)}^{\mu\nu\rho}$  that select the appropriate contraction and symmetry structure for each field type, an avenue left open for future formal development.

### C. Interpretation and consistency with General Relativity

The field components  $\mathcal{G}^{(X)}$  can be seen as mode projections of a symmetric deformation tensor  $\mathcal{G}_{\mu\nu}$ , whose full dynamics, in the relativistic regime, would be governed by a generally covariant action:

$$S = \int \mathcal{L}_X \sqrt{-g} d^4x, \quad (144)$$

with  $\mathcal{L}_X$  defined for each scalar excitation or composite tensorial contraction. For instance, the Einstein–Hilbert action

$$S_{EH} = \frac{c^4}{16\pi G} \int R \sqrt{-g} d^4x, \quad (145)$$

can be seen as a special realization where the Lagrangian is given by the Ricci scalar  $R$  derived

from the underlying metric  $g_{\mu\nu}$ . To recover Einstein’s equations from the unified field dynamics, we could identify the macroscopic metric as a coarse-grained average:

$$g_{\mu\nu}(x) = \langle \mathcal{G}_{\mu\nu}(x) \rangle. \quad (146)$$

This averaging would define an emergent Riemannian geometry whose curvature satisfies  $G_{\mu\nu} = 8\pi G T_{\mu\nu}$  in the classical limit, and where  $\langle \cdot \rangle$  denotes a coarse-grained or averaged value of the tensor field over microscopic vacuum excitations. It does not necessarily imply a full quantum expectation value, but rather a macroscopic effective field akin to a thermodynamic mean, acting as the geometric background in which the dynamics of the elastic field unfold. The Einstein–Hilbert action would then describe the large-scale behavior of this emergent average, not the full tensor field  $\mathcal{G}_{\mu\nu}$  itself.

### D. Field Modes as Standing Waves of the Elastic Vacuum

Each excitation  $\mathcal{G}^{(X)}$  corresponds to a distinct deformation mode—radial or azimuthal—of the elastic vacuum. These modes behave as standing wave solutions of a field equation with spherical symmetry, and their spatial profiles naturally reflect this structure. Notably:

- The observed  $1/r$  dependence of physical fields (such as  $\vec{E}$ ,  $\vec{B}$ ,  $\vec{g}$ ,  $\vec{T}$ ) arises directly from the Green’s function of the Laplacian in three spatial dimensions [33], consistent with oscillatory responses in elastic and electromagnetic media.
- In the dimensional framework adopted here—where mass, charge, and temperature share the same fundamental dimension,  $[M] \equiv [Q] \equiv [T_{\text{emp}}] \equiv [L]$ —source terms such as mass or charge have dimension  $[L]$ , and thus expressions of the form *source*/ $r^2$  acquire the overall dimension  $[1/r]$ . This implies that the radial decay of fields such as  $\vec{E}$ ,  $\vec{g}$ , or  $\vec{T}$  is dimensionally equivalent to  $1/r$ , even when expressed in terms of a traditional source-over-distance-squared structure.
- Moreover, by identifying the physical fields directly with the scalar field response  $\mathcal{G}^{(X)}$  (rather than with its spatial derivatives), the model preserves the  $1/r$  behavior of the fields without invoking an explicit divergence or gradient operation. This stands in contrast to conventional formulations, where vector fields decay as  $1/r^2$  due to their derivative origin.

Despite the internal deformations encoded in  $\mathcal{G}_{\mu\nu}$ , Lorentz invariance is preserved at the level of the field equations. The field transforms covariantly, and the elastic vacuum is treated as a Lorentz-invariant medium whose excitations carry well-defined transformation properties under boosts and rotations. Thus, the elastic field paradigm maintains both internal geometric coherence and external compatibility with the fundamental symmetries of relativistic field theory, while offering a unified geometric origin for all classical fields and their characteristic falloffs.

### E. Outlook and Future Developments: Quantization and Gauge Extensions

Quantization of the unified field  $\mathcal{G}_{\mu\nu}$  can naturally proceed via a covariant path-integral formalism:

$$Z = \int \mathcal{D}\mathcal{G}_{\mu\nu} e^{iS[\mathcal{G}_{\mu\nu}]}, \quad (147)$$

where the action  $S$  is built from Lorentz-invariant scalars involving  $\mathcal{G}_{\mu\nu}$  and its derivatives. In this framework, each classical deformation mode  $\mathcal{G}^{(X)}$  corresponds to a quantized normal mode of the elastic vacuum, analogous to phonons in condensed matter or gauge bosons in standard field theory. The vacuum behaves as a lattice of coupled quantum oscillators whose eigenmodes give rise to the familiar field excitations. This path-integral formulation provides a consistent platform for computing quantum amplitudes and propagators, and suggests that classical fields emerge as expectation values or coherent states of quantized elastic modes.

#### Modal Symmetries and Gauge Generalizations

While the present work focuses on classical modes corresponding to electromagnetic, gravitational, and thermo-entropic phenomena, further developments may incorporate internal symmetries and non-Abelian structures. In particular, *gauge fields*—such as those of the weak and strong interactions—could emerge as *internal connections or curvature forms* associated with symmetry groups acting on internal indices of  $\mathcal{G}_{\mu\nu}$ , or through matrix-valued generalizations of the field (e.g.,  $\mathcal{G}_{\mu\nu}^a$  with gauge index  $a$ ).

A crucial first step in this direction is to connect the framework to the Standard Model. We propose that the Higgs field,  $\phi$ , can be identified with a projection of the unified field, such as its scalar component:  $\phi \propto \mathcal{G}_{00}$ . Under this

identification, electroweak symmetry breaking and the acquisition of mass by fermions would be emergent phenomena arising from the vacuum's response to a purely compressional deformation. This implies that the remaining components of the tensor  $\mathcal{G}_{\mu\nu}$  (such as  $\mathcal{G}_{0i}$  and the spatial components  $\mathcal{G}_{ij}$ ) necessarily represent new physics beyond the Standard Model, corresponding to the torsional (electromagnetic) and shear (gravito-entropic) modes of this unified substrate.

Likewise, *Fermionic matter fields* might arise via supersymmetric extensions of the framework, in which  $\mathcal{G}_{\mu\nu}$  is embedded in a superfield whose components include spinorial partners. This opens the possibility of viewing matter as a localized defect or topological excitation in the elastic substrate, governed by the same underlying dynamics.

#### Path Forward

These generalizations, while beyond the scope of the present paper, are structurally compatible with the elastic field paradigm and its modal decomposition. A fully developed theory would entail:

- Construction of the complete action functional  $S[\mathcal{G}_{\mu\nu}]$  including source couplings, curvature terms, and possibly non-linearities;
- Identification of symmetry groups associated with different field sectors (Abelian, non-Abelian, supersymmetric);
- Quantization via canonical or path-integral methods, and the study of resulting propagators and interactions;
- Exploration of topological solutions and their identification with particle-like excitations.

Thus, while the present work lays the geometric and dynamic foundations, a richer landscape of quantum and gauge-theoretic structures awaits development within the same unified elastic framework.

## Part VI: Cosmological and subatomic implications of the established framework

### XXIII. THE COSMOLOGICAL CONSTANT AS A THERMO-ENTROPIC PROPERTY OF THE VACUUM

#### A. The Geometric Structure of the Cosmological Constant

The Einstein field equation in its most general form, including the cosmological constant  $\Lambda$ , is:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (148)$$

When there is no matter or conventional energy present, i.e.,  $T_{\mu\nu} = 0$ , the Einstein field equation reduces to:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0 \quad (149)$$

In this case,  $\Lambda$  can be interpreted as a form of *intrinsic energy* of the vacuum, which acts as a source of spacetime curvature. This vacuum energy is present even in the absence of matter or radiation.

To describe the vacuum energy as a form of energy affecting the curvature of spacetime, we can reinterpret the term  $\Lambda g_{\mu\nu}$  as contributing to an *effective energy-momentum tensor* for the vacuum energy. This gives us the following form for the vacuum energy-momentum tensor:

$$T_{\mu\nu}^{\text{vac}} = -\frac{\Lambda c^4}{8\pi G}g_{\mu\nu} \quad (150)$$

This term behaves like a *perfect fluid* with a constant energy density  $\rho_{\text{vac}}$  and an associated pressure  $p_{\text{vac}}$  related to the vacuum energy. The vacuum energy behaves like a fluid with *negative pressure*, meaning the pressure  $p_{\text{vac}}$  is equal to  $-\rho_{\text{vac}}c^2$ .

Then, the relationship between  $\rho_{\text{vac}}$  and  $\Lambda$  can be obtained by identifying the term describing vacuum energy in the Einstein field equation with the standard form of a perfect fluid in cosmology. In a universe dominated by vacuum energy, the effective energy density can be expressed as:

$$\rho_{\text{vac}}c^2 = \frac{\Lambda c^4}{8\pi G} \quad (151)$$

Operating, one has that

$$4\pi G\rho_{\text{vac}} = \frac{1}{2}\Lambda c^2 \quad (152)$$

which shows how the cosmological constant  $\Lambda$  is fundamentally tied to the gravitational flux as an expression of Gauss Law, with vacuum energy

density  $\rho_{\text{vac}}$ , and a structure reminiscent of kinetic energy or Einstein's mass-energy equivalence formula. The right-hand side of the equation implies that  $\Lambda$  can be viewed as a scaling factor for the intrinsic gravitational flux associated with the vacuum.

From a thermodynamic perspective, this formulation resonates with the idea that gravity emerges from microscopic degrees of freedom, as suggested by holographic and entropic gravity approaches [52]. The cosmological constant in this context can be interpreted as a measure of the equilibrium state of the vacuum, analogous to the way that temperature regulates thermodynamic systems. This perspective aligns with Jacobson's derivation of Einstein's equations from thermodynamic principles [10], where fluctuations in vacuum energy sustain an equilibrium that manifests macroscopically as gravitational dynamics.

Solving for  $\Lambda$ , we have that:

$$\Lambda = \frac{8\pi G\rho_{\text{vac}}}{c^2} \quad (153)$$

Recall that, within our framework, effective vacuum energy density  $\rho_{\text{vac}}$  can be expressed (XIII A) as:

$$\rho_{\text{vac}} \approx \frac{\hbar c}{2\pi \cdot 1 \text{ m}^4} \quad (154)$$

Substituting (154) into (153), we obtain:

$$\Lambda \approx \frac{8\pi G}{c^2} \cdot \frac{\hbar c}{2\pi \cdot 1 \text{ m}^4} = \frac{4G\hbar}{c \cdot 1 \text{ m}^4} \quad (155)$$

Dividing both sides by 4, we arrive at:

$$\frac{\Lambda}{4} = G \cdot \frac{\hbar}{c \cdot 1 \text{ m}^4} \quad (156)$$

Finally, substituting  $\hbar \equiv \frac{1 \text{ m}^2}{c^4}$  from (23) and  $G \equiv \frac{1}{16\pi c}$  from (82), we arrive at:

$$\Lambda \equiv \frac{1}{4\pi c^6 \cdot 1 \text{ m}^2} \quad (157)$$

#### B. Thermo-Entropic Action Density and the Geometric Structure of the Cosmological Constant

Recalling that the thermo-entropic modal action was shown to be  $S_{\text{th}} = \frac{\hbar}{c^2}$ , we see that

$$\Lambda = \frac{S_{\text{th}}}{4\pi \cdot 1 \text{ m}^4}$$

This reveals the cosmological constant as the manifestation of a universal *modal action density per 4D volume*. It acts as a tension-like Lagrangian density of the vacuum, coupling entropy and expansion across a fundamental volume cell.

*Interpretation as Geometric Surface Tension*

Indeed, note that setting  $r = c^3 \cdot 1 \text{ m}$  situates  $\Lambda$  as an effective curvature density, with  $4\pi r^2 = 4\pi c^6 \cdot 1 \text{ m}^2$  representing the "surface" of an expanding spherical volume. Thus, the cosmological constant acquires a direct geometrical interpretation as an *inverse areal curvature density*, analogous to curvature or density of a spherical boundary in expanding space, projecting a constant action flux over the expanding boundary of the universe. From this viewpoint,  $\Lambda$  is not a bulk energy density but a quantized surface effect—a geometric relic of the thermo-entropic elasticity of the vacuum.

This form provides a physical interpretation in which the large-scale expansion of the universe is driven by a steady energy flow that distributes itself over the expanding boundary, dynamically adjusting the effective curvature density as the volume of the universe grows. This interpretation not only aligns with the curvature requirements of an accelerating universe but also positions  $\Lambda$  as a fundamental invariant describing how the vacuum tension distributes minimal action quanta across areal elements, reinforcing the idea that the cosmological constant is, in essence, the *vacuum's curvature Lagrangian*.

*Modal Structure of  $\Lambda$ : Field Strength Interpretation and Action Principle*

The appearance of the factor  $\frac{1}{4}$  in the expression for  $\Lambda$  [Eq. (156)] is not accidental—it matches the canonical structure of kinetic terms in gauge field Lagrangians, where the field strength is contracted to form a scalar:

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (158)$$

Interestingly, the right-hand side of (156) can be interpreted as an expression akin to the equipartition theorem. Here,  $G \equiv k_B \cdot c^2 \cdot \alpha^2$  plays the role of a scaled Boltzmann constant  $k_B$ , and  $\frac{\hbar}{c \cdot 1 \text{ m}^4}$  represents the energy density associated with the thermo-entropic field (XVIII A). Thus, we can interpret that  $\frac{\Lambda}{4}$  encodes the average energy density of the thermo-entropic field, distributed over the degrees of freedom of the spacetime lattice. This interpretation aligns with the idea that the cosmological constant arises from the quantum fluctuations of the vacuum, as predicted by quantum field theory.

Motivated by this analogy, we propose an effective thermo-entropic field strength tensor

$\mathcal{F}_{\mu\nu}^{(GE)}$ , derived from the projected modes of the symmetric deformation field  $\mathcal{G}_{\mu\nu}$  (143):

$$\mathcal{F}_{\mu\nu}^{(GE)} := \partial_\mu \mathcal{G}_\nu^{(T)} - \partial_\nu \mathcal{G}_\mu^{(T)}, \quad (159)$$

where  $\mathcal{G}_\mu^{(T)}$  represents an effective vector field associated with thermo-entropic modal projections of the vacuum (e.g., torsional or volumetric oscillations). The corresponding Lagrangian density for the thermo-entropic sector takes the form:

$$\mathcal{L}_{\text{GE}} = -\frac{1}{4}\mathcal{F}_{\mu\nu}^{(GE)}\mathcal{F}^{(GE)\mu\nu}, \quad (160)$$

which naturally integrates into the total vacuum action. In this picture, the cosmological constant  $\Lambda$  arises as the average contraction:

$$\Lambda = \langle \mathcal{L}_{\text{GE}} \rangle, \quad (161)$$

interpreted as the *modal energy density* of the vacuum associated with thermo-entropic fluctuations.

This unifies a geometric view of  $\Lambda$  as spacetime curvature with a field-theoretic interpretation grounded in tensor dynamics. In this framework, the cosmological constant ceases to be an arbitrary constant and instead emerges as a vacuum-averaged scalar—encoding the net effect of thermo-entropic field fluctuations projected from the underlying symmetric structure of spacetime.

Moreover, since the modal action density of the thermo-entropic field was shown to scale as  $S_{\text{th}} = \hbar/c^2$ , the appearance of  $\Lambda$  as a contraction of field strength terms suggests that each minimal spacetime cell contributes a quantized action unit to the curvature of the vacuum. The cosmological constant thus acts as a macroscopic residue of these microscopic modal interactions—a kind of elastic equilibrium tension that encapsulates the thermodynamic state of the vacuum in a covariant formulation- and the effective Lagrangian density of an emergent field theory, derived from the symmetry-reduced oscillatory modes of an elastic and quantized vacuum substrate.

*The Einstein-Hilbert Action and the Fundamental Action of the Thermo-Entropic Field*

The reduction of the Einstein-Hilbert action in a vacuum-dominated universe to the form (103)

$$S_{\text{EH}} = \frac{c^4}{8\pi G} \int \Lambda \sqrt{-g} d^4x \quad (162)$$

reveals a profound identity when combined with our identification of the cosmological constant  $\Lambda$

as the effective Lagrangian density of the thermo-entropic field,  $\Lambda = \langle \mathcal{L}_{GE} \rangle$ . Substituting this identification back into the Einstein-Hilbert action, we obtain:

$$S_{EH} = \frac{c^4}{8\pi G} \int \langle \mathcal{L}_{GE} \rangle \sqrt{-g} d^4x \quad (163)$$

This result has fundamental implications for the nature of gravity. It demonstrates that the *Einstein-Hilbert action is, up to the prefactor, identical to the effective action of the thermo-entropic field*. The dynamics of spacetime geometry are thus shown to be a direct macroscopic manifestation of the underlying thermodynamics of the vacuum.

This framework provides a concrete physical basis for the emergent gravity paradigm. The principle of least action for geometry ( $\delta S_{EH} = 0$ ) can be reinterpreted as a principle of extremal entropic action. Spacetime evolves in such a way as to extremize the total action of its own underlying thermo-entropic fluctuations. In this view, the prefactor  $\frac{c^4}{16\pi G}$  is no longer merely a coupling constant for gravity, but rather the fundamental conversion factor that translates the action of the thermo-entropic field—a quantity rooted in thermodynamics and quantum fluctuations—into the language of classical, geometric curvature. This directly realizes the vision hinted at by Jacobson and Verlinde [10] [11], where the laws of gravity are emergent from the thermodynamics of a deeper substrate, which this framework identifies as the Quantum Oscillator Lattice.

#### XXIV. THE SCALE-DEPENDENT GRAVITATIONAL COUPLING AND THE RESOLUTION OF COSMOLOGICAL TENSIONS

We have previously shown (see Section XIB) how the gravitation constant exhibits a scale-dependent behavior. This scale dependence on some effective gravitational constant  $G_{\text{eff}}(z)$  is proposed as the underlying physical mechanism unifying the explanation of several major cosmological puzzles, including the *Hubble tension*, the *dark sector* phenomena, and the *growth of structure tension*, as will be detailed in the following sections.

##### A. A Phenomenological Model for the scale-dependent gravitational constant

For describing the continuous transition of the effective gravitational constant  $G_{\text{eff}}(z)$ , a physically well-motivated and universally recognized

function is the classic logistic (or sigmoid) function:

$$G_{\text{eff}}(z) = G_{\text{Glob}} + \left( \frac{G_{\text{Loc}} - G_{\text{Glob}}}{1 + e^{\alpha(z-z_c)}} \right) \quad (164)$$

Here,  $G_{\text{Loc}}$  and  $G_{\text{Glob}}$  are the asymptotic boundary values of the gravitational coupling.  $z_c$  represents the transition redshift (the cosmic epoch at which the transition is centered), corresponding to the era when the Universe's large-scale structure shifted from being dominated by linear perturbations to being characterized by non-linear, gravitationally collapsed objects. Based on standard cosmological models, a value in the range of  $z_c \approx 1 - 3$  would be physically expected [53]. Finally,  $\alpha$  represents the transition rate, dictating the steepness or duration of the transition. A large  $\alpha$  implies a rapid shift between the two gravitational regimes over a narrow redshift range, whereas a small  $\alpha$  suggests a more gradual evolution.

This phenomenological model is not merely a fitting function; it provides a concrete framework for falsifying or validating our theory. By constraining the free parameters  $z_c$  and  $\alpha$  using a combination of cosmological data—such as Type Ia Supernovae (SN Ia), Baryon Acoustic Oscillations (BAO), and Redshift-Space Distortions (RSD) [54, 55]—we can test whether a single, physically plausible transition can simultaneously explain the observed values of  $H(z)$ , the growth of structure, and other cosmological probes. A successful fit would provide compelling evidence for the scale-dependent nature of gravity proposed in this work.

##### B. Self-Interaction Terms as the Source of Hubble Tension

The plausibility of our hypothesis can be checked using Friedmann equations [56] [57] [58]. The first Friedmann equation is given by:

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_{\text{vac}} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \quad (165)$$

This equation relates the rate of expansion (the Hubble parameter,  $H = \dot{a}/a$ ) to the energy density of the universe. Assuming a nearly flat universe ( $k \approx 0$ ), the Hubble parameter can be calculated as

$$H^2 = \frac{8\pi G}{3} \rho_{\text{vac}} + \frac{\Lambda c^2}{3},$$

Substituting with our previous expression for  $\Lambda$  (153), we have that

$$\begin{aligned} H^2 &= \frac{8\pi G}{3} \rho_{\text{vac}} + \frac{\Lambda c^2}{3} = \\ &\frac{8\pi G}{3} \rho_{\text{vac}} + \frac{8\pi G \frac{\rho_{\text{vac}}}{c^2} c^2}{3} = \\ 2 \left( \frac{8\pi G}{3} \rho_{\text{vac}} \right) &= \frac{16\pi G}{3} \rho_{\text{vac}} \end{aligned}$$

Now, two regimes emerge naturally. Using  $\rho_{\text{vac}} = \frac{\hbar c}{2\pi \cdot 1 \text{ m}^4}$  as we have derived for the electromagnetic field (XIII A), we have that:

### 1. Global Regime (no self-interactions):

$$\begin{aligned} H_{\text{Glob}} &= \sqrt{\frac{16\pi G_{\text{Glob}}}{3} \rho_{\text{vac}}} \\ &= \sqrt{\frac{16\pi \cdot 2\pi \varepsilon_0}{3} \frac{\hbar c}{2\pi \cdot 1 \text{ m}^4}} \\ &\approx 2.165 \times 10^{-18} \text{ s}^{-1} = 66.81 \text{ km/s/Mpc} \end{aligned} \quad (166)$$

This matches the CMB-based Planck 2018 measurement:  $H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc}$  [59].

### 2. Local Regime (includes self-interactions):

$$\begin{aligned} H_{\text{Loc}} &= \sqrt{\frac{16\pi G_{\text{Loc}}}{3} \rho_{\text{vac}}} \\ &= \sqrt{\frac{16\pi \cdot \frac{3}{5} 4\pi \varepsilon_0}{3} \frac{\hbar c}{2\pi \cdot 1 \text{ m}^4}} \\ &\approx 2.3714 \times 10^{-18} \text{ s}^{-1} = 73.17 \text{ km/s/Mpc} \end{aligned} \quad (167)$$

This matches the SH0ES result:  $H_0 = 73 \pm 1.0 \text{ km/s/Mpc}$  [60].

## C. Discussion and Observational Tests

Our framework predicts that the Hubble parameter  $H(z)$  evolves between two fixed values due to a scale-dependent gravitational constant:

$$\begin{aligned} \lim_{z \rightarrow 0} H(z) &\rightarrow H_{\text{Loc}} \approx 73 \text{ km/s/Mpc} \\ \lim_{z \rightarrow \infty} H(z) &\rightarrow H_{\text{Glob}} \approx 67 \text{ km/s/Mpc} \end{aligned} \quad (168)$$

This interpretation leads to several concrete, testable consequences:

- **Redshift evolution:** As self-interaction effects dominate locally, we expect  $G_{\text{eff}}(z) \approx G_{\text{Loc}}$  at low redshifts, yielding a higher inferred Hubble constant  $H(z \approx 0) \approx 73 \text{ km/s/Mpc}$ . At higher redshifts—probing

smoother, linear regimes— $G_{\text{eff}}(z) \rightarrow G_{\text{Glob}}$ , driving  $H(z)$  down toward  $\sim 67 \text{ km/s/Mpc}$ . A continuous transition in  $H(z)$  would strongly support our hypothesis.

- **Structure formation diagnostics:** A discrepancy between local and early-universe structure growth would corroborate scale-dependent gravitational dynamics.

## D. Theoretical Context and Consistency

The presented model serves as an *effective field theory*: standard General Relativity with Newton's constant  $G_N$  describes gravity accurately in the local, nonlinear regime (i.e.,  $G_{\text{Loc}} \equiv G_N$ ), while on cosmological scales, averaging over large volumes under homogeneity and isotropy leads to effective dynamics that resemble General Relativity but with a renormalized, scale-dependent coupling  $G_{\text{Glob}}$ . In this perspective, the Friedmann equations remain applicable because they emerge from the Einstein field equations under symmetry assumptions, and the modification resides not in the geometry, but in the coupling between geometry and energy-momentum.

## E. Scale-Dependent Vacuum Energy and Implications for the Dark Sector

As derived in previous sections (157), we obtain a geometric expression for the cosmological constant:

$$\Lambda = \frac{1}{4\pi c^6 \cdot 1 \text{ m}^2}$$

Using the standard expression that relates the vacuum energy density to the cosmological constant:

$$\rho_{\text{vac}} = \frac{\Lambda c^2}{8\pi G}$$

we see that  $\rho_{\text{vac}}$  is inversely proportional to the gravitational coupling  $G$ . Since our framework posits that  $G$  is scale-dependent—shifting between  $G_{\text{Loc}}$  and  $G_{\text{Glob}}$  depending on the cosmic regime—then, *if  $c$  is constant, we have that  $\rho_{\text{vac}}$  is necessarily scale-dependent as well.*

### Implications for Dark Energy

In our framework, the accelerated expansion of the Universe is driven by the fundamental cosmological constant  $\Lambda$ , which is a geometric invariant of the vacuum structure as derived in Eq. (157). However, the *effective vacuum energy density*,  $\rho_{\text{vac}}$ , which is often used to interpret this

phenomenon, becomes a scale-dependent quantity due to its relationship with  $G$ :

$$\rho_{\text{vac}}(z) = \frac{\Lambda c^2}{8\pi G_{\text{eff}}(z)}$$

This leads to a profound reinterpretation of the "dark energy problem". The problem is often framed as two questions: why is  $\rho_{\text{vac}}$  not zero, and why is its value so small compared to theoretical predictions from particle physics? Our model offers a new perspective:  $\rho_{\text{vac}}$  is not an independent entity but is fundamentally tied to the gravitational coupling.

The observed cosmic acceleration is not caused by a mysterious, constant fluid, but is an intrinsic feature of spacetime geometry encoded by  $\Lambda$ . The "dark energy component" of the  $\Lambda$ CDM model, with its fine-tuned value, can be seen as an artifact of assuming a constant  $G_N$ . Our framework provides a direct physical link between the vacuum energy density and the elastic properties of spacetime, suggesting that its measured value is a necessary consequence of the Universe's gravitational state at cosmological scales.

#### *Implications for Dark Matter*

The dark matter problem also finds a potential reinterpretation under this framework. Gravitational analyses of galaxies and clusters typically assume a uniform  $\rho_{\text{vac}}$  based on global (CMB-scale) fits. However, these local systems operate under a different gravitational regime—characterized by  $G_{\text{Loc}}$ , and hence a smaller vacuum energy density:

$$\rho_{\text{vac}}^{\text{loc}} = \frac{\Lambda c^2}{8\pi G_{\text{Loc}}} < \rho_{\text{vac}}^{\text{glob}}$$

Specifically, the ratio of the "true" local vacuum energy density to the globally-inferred one is:

$$\frac{\rho_{\text{vac}}^{\text{loc}}}{\rho_{\text{vac}}^{\text{glob}}} = \frac{G_{\text{Glob}}}{G_{\text{Loc}}} \approx 0.833$$

This means that when cosmologists use a global fit to estimate the energy budget of a local structure, they are systematically overestimating the contribution of vacuum energy by approximately 20%. This overestimation directly leads to a miscalculation of the required matter content to explain the observed dynamics.

This misestimate can lead to an apparent deficit in the gravitational binding, prompting the postulation of unseen mass (i.e., dark matter). When the correct local value of  $\rho_{\text{vac}}$  is applied,

the gravitational field is stronger than previously inferred from global fits, reducing or possibly eliminating the need for dark matter in certain contexts.

#### *Summary of the Unified Picture*

In this unified view, both dark energy and dark matter effects are linked to the same underlying principle: the scale-dependence of gravitational coupling  $G$ , and thus the vacuum energy density  $\rho_{\text{vac}}(z)$ . The former (dark energy) emerges from the global underestimation of gravitational binding in the smooth Universe, while the latter (dark matter) arises from the overestimation of vacuum energy at small scales.

*Rather than invoking unknown forms of matter or energy, this framework attributes the dark sector phenomenology to misinterpretations arising from applying a constant gravitational coupling across regimes where gravity is fundamentally scale-sensitive.*

The observational consequences of this reinterpretation remain testable through precision cosmology, particularly via redshift evolution of  $H(z)$ , structure growth, and gravitational lensing signatures—offering a compelling alternative to the standard  $\Lambda$ CDM paradigm. In the following section, we demonstrate that the explanatory power of this framework extends beyond the dark sector to other key tensions in modern cosmology.

#### **F. A Unified Framework for Other Cosmological Tensions**

Our framework possesses an explanatory power that extends to several other significant discrepancies between the early and late Universe. This section demonstrates how the two gravitational coupling scales derived from our model,

$$G_{\text{Loc}} = \frac{3}{5}4\pi\epsilon_0 \quad \text{and} \quad G_{\text{Glob}} = 2\pi\epsilon_0, \quad (169)$$

offer a unified physical origin for the tension in the structure growth parameter ( $S_8$ ), and anomalies in gravitational lensing.

#### *The Growth of Structure Tension ( $S_8$ )*

One of the most relevant tensions in modern cosmology, aside from  $H_0$ , concerns the  $S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$  parameter, which quantifies the amplitude of matter fluctuations on large

scales. Measurements from the Cosmic Microwave Background (CMB) by the Planck satellite, which probe the early Universe, predict a significantly higher value for  $S_8$  than what is measured directly in the local Universe via weak gravitational lensing and galaxy cluster surveys (Large-Scale Structure, LSS) [61].

Our framework naturally resolves this discrepancy. The growth of density perturbations is directly governed by the value of the gravitational constant. In our model:

1. The true growth of large-scale structure is governed by the weaker global coupling,  $G_{\text{Glob}}$ .
2. Standard  $\Lambda$ CDM predictions, however, are derived from a model calibrated against local physics, where the gravitational constant is assumed to be the stronger  $G_N \equiv G_{\text{Loc}}$ .

Since the global coupling is approximately 17% weaker than the local one ( $G_{\text{Glob}}/G_{\text{Loc}} \approx 0.833$ ), the rate of structure formation is naturally suppressed over cosmic time compared to the standard prediction. The lower value of  $S_8$  observed in LSS surveys is thus an expected consequence of gravity being intrinsically weaker on the vast, homogeneous scales these surveys probe. The tension arises from the attempt to reconcile a prediction based on a strong gravity model with an observation occurring in a weak gravity regime.

#### *Systematic Discrepancies in Gravitational Lensing*

The phenomenon of gravitational lensing provides a crucial test for the scale-dependence of  $G$ .

- **Strong Lensing** occurs in the dense, compact cores of galaxies and clusters (kpc scales), a decidedly local regime governed by  $G_{\text{Loc}}$ .
- **Weak Lensing** measures the subtle distortion of background galaxies by the matter distribution on much larger scales (Mpc), averaging over vast cosmic volumes. This regime is therefore dominated by  $G_{\text{Glob}}$ .

This dichotomy leads to a concrete, falsifiable prediction: if an observer assumes a constant  $G = G_N \equiv G_{\text{Loc}}$  to infer mass from a weak lensing signal, they will systematically underestimate the true mass. The inferred mass ( $M_{\text{inf}}$ ) will relate to the real mass ( $M_{\text{real}}$ ) via the ratio of the couplings:

$$M_{\text{inf-weak}} = M_{\text{real}} \cdot \frac{G_{\text{Glob}}}{G_{\text{Loc}}} \approx 0.833 \cdot M_{\text{real}} \quad (170)$$

This predicted 17% systematic mass deficit in weak lensing analyses (under the assumption of a constant  $G$ ) is a unique signature of our framework.

## XXV. THE EMERGENCE OF FUNDAMENTAL PARTICLES FROM THE VACUUM'S STRUCTURE

A core test for any unified framework is its ability to derive the properties of fundamental particles from its foundational principles. In this section, we demonstrate that the electron mass,  $m_e$ , is not a fundamental constant but rather an emergent property of the vacuum. We derive a predictive formula for  $m_e$  that arises directly from a hypothesized link between the quantum atomic scale and the effective cosmological parameters established within this theory.

### A. Derivation of the Electron Mass from the Atomic-Cosmological Scale Correspondence

The structure of the atom is defined by the Bohr radius,  $a_0$ , which represents the most probable orbital distance of an electron in its ground state. Conventionally, it is given by the balance between quantum mechanics and electromagnetism:

$$a_0 = \frac{\hbar}{m_e c \alpha} \quad (171)$$

In our framework, we advance a central hypothesis that connects this micro-physical scale to the macro-physical properties of the vacuum. We postulate that the Bohr radius emerges directly from the dielectric structure of the vacuum, scaled by the characteristic length of the underlying vacuum oscillator:

$$a_0 = 2\pi\epsilon_0 \cdot 1 \text{ m} \quad (172)$$

This expression identifies the fundamental atomic scale as a natural unit of length derived from vacuum elasticity alone, thereby grounding quantum structure in the geometric-dielectric properties of space.

By equating these two expressions for the Bohr radius, we can test the consistency of our framework and derive a new expression for the electron mass. This is not merely an algebraic manipulation, but a profound statement that the inertia of the electron must be precisely what is required to satisfy this micro-macro scale correspondence.

$$\frac{\hbar}{m_e c \alpha} = 2\pi\epsilon_0 \cdot 1 \text{ m} \quad (173)$$

Solving for the electron mass,  $m_e$ , yields the following predictive formula:

$$m_e = \frac{\hbar}{2\pi\epsilon_0 \cdot 1 m \cdot c \cdot \alpha} \quad (174)$$

This equation is a key result of our theory. It asserts that the electron's mass is entirely determined by the fundamental constants governing the vacuum's quantum ( $\hbar$ ), electromagnetic ( $\epsilon_0, \alpha, c$ ), and geometric properties.

Evaluating this expression using the accepted CODATA values for the constants results in a predicted mass of  $m_e \approx 8.66 \times 10^{-31}$  kg. This value matches the experimentally measured electron mass ( $9.11 \times 10^{-31}$  kg) to within 5%, with the small discrepancy attributable to second-order effects or radiative corrections not included in this leading-order derivation. The remarkable accuracy of this result provides strong quantitative support for the foundational hypothesis stated in Equation 172.

1. *Consistency with the constitutive elasticity law*  
 $m = -kS$ .

Having derived the electron mass from the atomic-vacuum correspondence, we now demonstrate its profound consistency with the other foundational principles of this theory. From the constitutive relation  $m = kS$ , previously derived as a generalized Hookean response of the vacuum (see Eq. 60), we introduce the concept of a fundamental vacuum mass quantum,  $m_{\text{vac}}$ , which characterizes the inertial response of the vacuum to unit entropic action. Taking  $k \equiv \frac{1}{C} = \frac{1}{\epsilon_0 \cdot 1 m}$  and the quantum of action as the modal electromagnetic action  $S = \hbar \equiv \frac{1 m^2}{c^4}$  (23), we obtain:

$$m_{\text{ref}} \equiv \frac{\hbar}{\epsilon_0 \cdot 1 m} \equiv \frac{\mu_0^2 \epsilon_0^2 \cdot 1 m^2}{\epsilon_0 \cdot 1 m} \equiv k_B \cdot 1 K \quad (175)$$

where we have used the equivalences  $k_B \equiv \frac{\mu_0}{c^2} \equiv \mu_0^2 \epsilon_0$  and  $1 m \equiv 1 K$ .

This value represents a fundamental reference mass scale encoded in the elastic structure of the vacuum, and establishes a profound thermodynamic correspondence. It demonstrates that the reference mass of the vacuum—a quantity emerging from the interplay between quantum action and the vacuum's elastic compliance—is fundamentally identical to the characteristic unit of thermal energy. This identity suggests that *mass, at its most fundamental level, is intrinsically determined by the thermodynamic capacity of the vacuum to host energy in its*

*degrees of freedom*. In essence, the vacuum's resistance to deformation and its ability to mediate thermal interactions are two manifestations of the same underlying geometric and entropic structure.

Comparing this fundamental vacuum mass with the electron mass derived in Eq. (174), we find the strikingly simple relation:

$$m_e \approx \frac{m_{\text{ref}}}{2\pi \cdot c\alpha} \propto m_{\text{ref}} \cdot \zeta^3$$

where we have used that  $c \propto \alpha^{-4}$  (100).

This result provides a deep physical insight into the nature of matter. It suggests that the electron's mass ( $m_e$ ) is not an intrinsic property but a manifestation of the vacuum's fundamental thermo-massive scale ( $m_{\text{ref}}$ ), modulated by a non-linear self-interaction factor. The proportionality to the cube of the damping ratio ( $\zeta^3$ ) indicates that the formation of a stable particle like the electron is not a simple *dressing* of the vacuum's base excitation, but rather a higher-order, volumetric self-damping process. In this interpretation, the electron's mass emerges as a measure of the intensity of the dissipative self-interaction required to "precipitate" a coherent, localized excitation from the underlying vacuum field, consolidating the view of particles as stable topological states of the spacetime structure itself, strengthening both the internal coherence and the predictive power of the framework.

2. *Conclusions*

As a result, in this framework, the electron is no longer a fundamental point particle but a *stable, localized excitation* of the vacuum's oscillator lattice. Its observed inertial mass is not predefined but emerges from how the vacuum oscillators couple to and respond to the embedded electromagnetic deformation and relativistic effects. We have just shown again (23) how the equation  $m = -kS$  captures the essence of mass as a geometric-vacuum response property: it expresses mass as the effective resistance of the vacuum to entropic displacement, where  $k$  quantifies the vacuum's elastic stiffness and  $S$  encodes the action required to deform it. In this sense, inertial mass is not an intrinsic attribute, but a measure of how much action is stored—and resisted—by the vacuum structure under electromagnetic excitation.

This perspective parallels contemporary ideas in quantum field theory, where inertial mass arises through field interactions (e.g., via the

Higgs mechanism). Here, however, the vacuum structure is modeled as a coherent assembly of harmonic oscillators whose coupling gives rise to mass. It supports the broader view that mass is not an immutable quantity but an emergent phenomenon tied to the local geometry and internal dynamics of the vacuum.

Finally, it is remarkable that the Bohr radius, as defined in our framework by  $a_0 = 2\pi\epsilon_0 \cdot 1 \text{ m}$ , shares the same geometric-dielectric factor that defines the global effective gravitational constant,  $G_{\text{Glob}} = 2\pi\epsilon_0$  (XIB). This common structure shows how both atomic-scale and cosmological-scale interactions can be governed by the same elastic properties of the vacuum, adding another powerful layer of consistency to our proposed framework.

## XXVI. FINAL CONCLUSIONS: A NEW PARADIGM FOR THE COSMOS

### A. The Nature of the Physical Substrate: An Elastic, Dissipative Quantum Vacuum

Throughout this work, we have treated the vacuum not as a passive, empty stage, but as the central actor from which all physical phenomena emerge. The consistency of our derivations allows us to precisely define the nature of this substrate. It is not a substance that "fills" space; rather, **it is space itself**, characterized by four fundamental, interlinked properties:

1. **A Unified Field.** The substrate is a single, unified field. The apparent distinction between gravity and electromagnetism is an illusion born of observing different *modes of vibration* of this medium. Longitudinal, compressional deformations manifest as gravity, while transverse, shear-like deformations manifest as electromagnetism. The forces of nature are thus unified not by a new particle, but by the common elastic origin of their dynamics.
2. **An Elastic Medium.** The substrate possesses intrinsic elastic properties, which we can quantify. Its resistance to different modes of deformation is what defines the fundamental constants. We have shown that the vacuum's immense transverse stiffness is quantified by the Coulomb constant ( $K_e$ ), while its corresponding longitudinal compliance—or softness—is quantified by the gravitational constant ( $G$ ). These are not arbitrary numbers, but the measured elastic moduli of spacetime.

3. **A Dissipative Medium.** The vacuum is not a perfect, frictionless elastic. It possesses an intrinsic "viscosity" or "damping," a property that is fundamental to its nature. This dissipation is quantified by the fine-structure constant,  $\alpha$ , through the universal damping ratio,  $\zeta = \alpha$ . We have demonstrated that this damping is the mechanism responsible for the emergence of both gravity and charge as second-order, attenuated "echoes" of the vacuum's primary properties. It is this intrinsic friction of spacetime that gives rise to the quantized resistance measured in condensed matter systems.

4. **A Quantum Medium.** Finally, the substrate is quantized. It is best visualized as a lattice of fundamental quantum oscillators. The discreteness of this lattice is governed by the quantum of action,  $\hbar$ . We have shown that the very existence and evolution of a fundamental 4-volume of this vacuum lattice corresponds to one unit of  $\hbar$ , thereby unifying the geometry of General Relativity with the granularity of the quantum world.

In summary, our theory describes a universe built upon a **quantized, elastic, and dissipative vacuum**. This single, unified entity provides the physical substrate whose properties give rise to all the laws and constants of nature as we know them.

### B. Correspondence with Formal Gauge-Theoretic Frameworks

There is a profound conceptual correspondence between the *Quantum Oscillator Lattice* (QOL) framework presented in this work and the formal gauge theory of *Unified Gravity* (UG) proposed by Partanen and Tulkki while this Paper was being finished [17]. While the former proceeds from physical analogies and dimensional postulates and the latter from the rigorous application of gauge theory, they converge on a remarkably similar vision of fundamental physics. We argue that the UG formalism can be interpreted as a potential mathematical realization of the physical principles established in the QOL theory, providing a pathway toward a complete, renormalizable quantum theory.

#### *Points of Conceptual Convergence*

- a. *The Unified Field as an Emergent Structure* A central tenet of the QOL theory is that all fundamental fields are emergent. They are described as distinct "modal projections" of a

single, symmetric rank-2 deformation tensor,  $\mathcal{G}_{\mu\nu}$ , which characterizes the state of an underlying elastic spacetime substrate. This concept finds a powerful mathematical analogue in the UG theory. Partanen and Tulkki’s framework achieves unification by placing gravity on the same footing as the Standard Model interactions. It derives gravity from a set of four  $U(1)$  gauge symmetries acting on a newly introduced “*space-time dimension field*”.

The correspondence is clear: *QOL’s “modal projections” can be seen as a physical description of what UG formalizes as distinct gauge symmetries.* Both frameworks posit that the perceived separation of forces is an emergent illusion, arising from the different ways a single, underlying structure can be excited or observed.

*b. The Origin of Gravity and Its Source* The QOL theory derives the gravitational constant  $G$  as an effective parameter describing the vacuum’s elastic stiffness to longitudinal deformations, relating it to electromagnetic and quantum parameters via  $G = \mu_0\alpha^2$ . Gravity is thus not fundamental, but a specific response of the vacuum lattice to mass-energy.

The UG theory provides a stunning formal justification for this view. Through the application of Noether’s theorem to its novel  $U(1)$  symmetries, it derives the symmetric stress-energy-momentum (SEM) tensor as the conserved current that sources the gravitational field. This is a pivotal result. While QOL postulates the coupling of gravity to mass-energy, UG derives this coupling from first principles of symmetry.

Therefore, one can argue that *the UG framework provides the formal gauge-theoretic reason why the vacuum’s elastic response (in QOL’s language) must be sourced by the SEM tensor.*

*c. The Nature of Spacetime and Renormalizability* Both theories challenge the classical view of spacetime as a passive background, and both are designed to be compatible with General Relativity (GR):

- The QOL theory suggests that the GR metric tensor  $g_{\mu\nu}$  is a coarse-grained average of the fundamental deformation tensor, i.e.,  $g_{\mu\nu}(x) = \langle \mathcal{G}_{\mu\nu}(x) \rangle$ . It further reinterprets mass-energy not as something that curves spacetime, but as being deformed spacetime itself.

- The UG theory demonstrates its classical consistency by showing that it can exactly reproduce the Teleparallel Equivalent of General Relativity (TEGR), which is known to be equivalent to GR. More radically, its UGM formulation allows for an exact description of gravity on a flat Minkowski metric, separating the interaction from the background geometry.

*d. The path to a quantum theory* The deepest correspondence lies in the path to a quantum theory. The QOL framework, through its dimensional collapse  $[M] \equiv [L] \equiv [T]$ , naturally leads to dimensionless constants. The UG theory achieves a dimensionless coupling constant for gravity, a necessary condition for renormalizability that conventional quantum gravity fails to meet. Partanen and Tulkki proceed to show that their theory is indeed renormalizable at one-loop order. *This suggests that the UG theory provides the explicit, renormalizable quantum field theory that the QOL framework intuits and sets the conceptual stage for.*

In conclusion, the QOL theory can be viewed as providing the physical and philosophical framework—the ‘*what*’—by postulating a unified elastic vacuum from simple principles. The UG theory, in turn, provides the rigorous mathematical machinery—the ‘*how*’—demonstrating that such a unification is not only possible within the established language of gauge theory but also resolves the critical issue of non-renormalizability. The success of the UG formalism lends significant theoretical support to the foundational postulates of the QOL model, suggesting they are not mere analogy but reflect a deep, symmetric structure of our universe.

### C. Final Thoughts

This model challenges our notions of what is fundamental in the universe. If gravity, electromagnetism, and quantum phenomena all arise from the same oscillatory vacuum, then the distinction between these forces are more illusory than real. They are just expressions of the same underlying reality, a vibrating cosmos that resonates through every level of existence—from the quantum realm to the largest cosmic structures.

The internal coherence of the relationships derived throughout this work hints at a deeper truth: that the complexity of the universe arises from simple, unified principles grounded in the oscillatory behavior of the vacuum. This realization suggests that the universe is not a fragmented

collection of forces and constants, but a deeply interconnected whole, where every phenomenon is an expression of the same underlying dynamics, and divisions between forces and fields are merely artifacts of our limited understanding, and where every aspect of reality is a manifestation of the same fundamental processes.

This model also resonates with the philosophical principle of simplicity, or "*Occam's Razor*", which suggests that the simplest explanation that accounts for all phenomena is likely to be correct. The notion that the universe's complexity—spanning from quantum mechanics to general relativity—can be fundamentally explained through the dynamics of vacuum oscillations provides a powerful example of how simplicity can reveal profound truths.

In summary, the metaphysical vision offered

as a byproduct by this model invites us to reconsider the nature of the universe as a whole. It suggests a cosmos that is not a static structure governed by immutable laws but a dynamic, evolving system where everything is interconnected. This invites a more holistic view of the cosmos, where complexity and diversity arise from simple, fundamental vibrations at the heart of reality itself.

### Declaration of generative AI and AI-assisted technologies in the writing process

*During the preparation of this work the author used Generative AI to improve the quality of the narrative of the Paper, and peer-review and check the internal consistency of the theoretical derivations. After using this tool/service, the author reviewed and edited the content as needed, and takes full responsibility for the content of the publication.*

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