Proof of Goldbach's conjecture and Twin prime number conjecture

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Abstract: Goldbach's conjecture has been around for more than 300 years and the twin prime conjecture for more than 160 years. Both remain unsolved. Both conjectures are important number theory conjectures for studying prime numbers. This article proposes a method of sequence shift to solve. This may be a good method and seems quite easy to understand.

Keywords: Goldbach's conjecture, conjecture of twin primes, prime numbers, even numbers.

1. Introduction:

Goldbach's conjecture

Any even number greater than 2 can be expressed as the sum of two prime numbers.

Conjecture of twin primes

There are infinite prime number pairs (p, p+2).

2. Proof Process

Proof:

First step: Prove that any even number greater than or equal to 2 can be expressed as the difference of two prime numbers.

Construct a sequence R that only contains prime numbers and sets all other numbers to 0; R(0)=0, R(1)=0.

R=[0,0,0,3,0,5,0,7,0,0,0,11, ...]

To prove the aforementioned proposition, it can be formulated as determining whether there exists a common position where both the original sequence R and the sequence R shifted right by M positions contain prime numbers.

$R(t)$ & $R(t+M) = 1$ ($t \in (R(t) \neq 0)$).

At this point, the even number M represents the difference between the two prime numbers located at the same position in the shifted sequence and the original sequence R.

Now, let's proceed with the proof using proof by contradiction：

N is an even number greater than or equal to 2, representing the difference between two prime numbers. In the correct case, it should be:

$R(t)\&R(t+2)=1$
$R(t)\&R(t+4)=1$
$R(t)\&R(t+6)=1$
...
$R(t)\&R(t+N+2)=1$
$R(t)\&R(t+N+4)=1$
$R(t)\&R(t+N+6)=1$
...

Each of the equations can find a number to satisfy when t takes values of $R(t) \neq 0$.

Below we prove that the above conditions $R(t)$ & $R(t+N) = 0$ and $R(t)$ & $R(t+N+2) = 1$, $R(t)$ & $R(t+N+4) = 1$, $R(t)$ & $R(t+N+6) = 1$, ... are impossible!

If $R(t)\&R(t+N)=0$
Then $R(t+2)\&R(t+N+2)=0$

the prime terms of R after a right shift by 2, and the terms corresponding to the prime terms of R,

$R(t)\&R(t+N+2)=0$

the prime terms of R after a right shift by 4, and the terms corresponding to the prime terms of R,

$R(t)\&R(t+N+4)=0$

the prime terms of R after a right shift by 6, and the terms corresponding to the prime terms of R,

$R(t)\&R(t+N+6)=0$

……

All the corresponding terms mentioned above refer to the zero terms in R after a right shift by 2 that correspond to the prime terms of R. This is because the zero terms in the R sequence after a right shift by N+4, the zero terms in the R sequence after a right shift by N+6, and so on, are all zero terms in the R sequence after a right shift by N+2.

Therefore

$R(t)\&R(t+N+2)=0$

By analogy

$R(t)\&R(t+N+4)=0$

$R(t)\&R(t+N+6)=0$

......

As mentioned above, if an even number N cannot be expressed as the difference of two prime numbers, then no even number greater than N can be expressed as the difference of two prime numbers either.

Second step: Prove that any even number greater than 2 can be expressed as the sum of two prime numbers.

Construct a sequence R that only contains prime numbers and sets all other numbers to 0; R(0)=0, R(1)=0.

R=[0,0,0,3,0,5,0,7,0,0,0,11, ...]

To prove the above proposition, it can be formulated as follows: if the sequence R is rotated and shifted right by M positions, the resulting sequence has the same prime numbers at the same positions as the original sequence R

$R(t) \& R(-t+M) = 1$ (t∈（R(t) ≠0),R(-t)=R(t)).

In this case, the even number M is the sum of the two prime numbers that are both prime at the same position in the rotated and translated sequence and the original sequence R.

Now, let's proceed with the proof using proof by contradiction：

N is an even number greater than 2, representing the sum of two prime numbers. In the correct case, it should be:

$R(t)\&R(-t+N-2)=1$

$R(t)\&R(-t+N-4)=1$

$R(t)\&R(-t+N-6)=1$

...

Each of the equations can find a number to satisfy when t takes values of R(t) ≠ 0.

Below we prove that the above conditions R(t) & R(-t+N) = 0 and R(t) & R(-t+N-2) = 1, R(t) & R(-t+N-4) = 1, R(t) & R(-t+N-6) = 1, ... are impossible!

If $\quad R(t)\&R(-t+N)=0$

Then $\quad R(t+2)\&R(-t+N-2)=0$

the prime terms of R after a right shift by 2, and the terms corresponding to the prime terms of R,

R(t)&R(-t+N-2)=0

the prime terms of R after a right shift by 4, and the terms corresponding to the prime terms of R,

R(t)&R(-t+N-4)=0

the prime terms of R after a right shift by 6, and the terms corresponding to the prime terms of R,

R(t)&R(-t+N-6)=0

……

All the corresponding terms mentioned above refer to the zero terms in R after a right shift by 2 that correspond to the prime terms of R. This is because the zero terms in the R sequence after rotation and a right shift by N-4, the zero terms in the R sequence after rotation and a right shift by N-6, and so on, are all zero terms in the R sequence after rotation and a right shift by N-2.

Therefore

R(t)&R(-t+N-2)=0

By analogy

R(t)&R(-t+N-4)=0

R(t)&R(-t+N-6)=0

......

As mentioned above, if an even number N greater than 2 cannot be expressed as the sum of two prime numbers, then no even number smaller than N can be expressed as the sum of two prime numbers either.

Third step: Prove that there are infinite prime number pairs (p, p+2).

Here, we also using the method of proof by contradiction**.**

Let's assume that there are only finite prime number pairs (p, p+2). Let U be a prime number greater than the largest prime pair p+2, and n∈(R(n) ≠0). At this point, we construct the sequence R=[U,0,...] as in the first step. Since there are no more prime number pairs (p, p+2), the sequence R shifted right by 2 will not have any prime numbers at the same positions as the original sequence R.

According to the derivation in the **First step**:

R(n)&R(n+2)=0

R(n)&R(n+4)=0

R(n)&R(n+6)=0

……

Prime numbers greater than U must necessarily lie at even positions that are right-shifted from U. However, as deduced above, no matter how much the sequence R is shifted to the right, there will never be prime numbers at the same positions simultaneously, indicating that the number of primes is finite!

Therefore, the aforementioned assumption is incorrect; that is, there still exist pairs of prime numbers with a difference of 2 after the prime number U. Hence, there are infinitely many pairs of primes p and p+2.

Fourth step: Generalize the above reasoning to (p, p+2k) based on the above derivation process.

Here, we also using the method of proof by contradiction**.**

Let's assume that there are only finite prime number pairs (p, p+2k). Let U be a prime number greater than the largest prime pair p+2k, and v∈（R(n) ≠0). At this point, we construct the sequence R=[U,0,…] as in the first step. Since there are no more prime number pairs (p, p+2k), the sequence R shifted right by 2k will not have any prime numbers at the same positions as the original sequence R. According to the derivation in the **First step：**

R(n)&R(n+2k)=0
R(n)&R(n+2k+2)=0
R(n)&R(n+2k+4)=0
……

Primes greater than U must necessarily occupy even positions that are right-shifted from U. However, as deduced above, no matter how much the sequence R is shifted to the right, there will never be primes at the same positions simultaneously, suggesting that the number of primes is finite!

Therefore, the aforementioned assumption is incorrect; that is, there still exist pairs of primes with a difference of 2k after the prime U, implying that there are infinitely many prime pairs of the form p and p+2k.

3. Conclusion

Based on the proofs above, the following conclusions can be drawn:

1.Any even number greater than or equal to 2 can be expressed as the difference of two prime numbers; if there exists an even number that cannot be expressed as the difference of two prime numbers, then all even numbers greater than that number also cannot be expressed in such a way.

2.Any even number greater than 2 can be expressed as the sum of two prime numbers; if there exists an even number greater than 2 that cannot be expressed as the sum of two prime numbers, then all even numbers smaller than that number also cannot be expressed in such a way.

3.There exist infinitely many pairs of primes (p, p+2); if there are only finitely many pairs of primes (p, p+2), then the total number of primes is finite.

4.There exist infinitely many pairs of primes (p, p+2k); if there are only finitely many pairs of primes (p, p+2k), then the total number of primes is finite.

4. References

The proof process in this paper is completely original and does not directly cite other literatures.