Proof of Goldbach's conjecture and Twin prime number conjecture

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Abstract: Goldbach's conjecture has been around for more than 300 years and the twin prime conjecture for more than 160 years. Both remain unsolved. Both conjectures are important number theory conjectures for studying prime numbers. This article proposes a method of sequence shift to solve. This may be a good method and seems quite easy to understand.

Keywords：Goldbach's conjecture, conjecture of twin primes, prime numbers, even numbers.

1. Introduction:

Goldbach's conjecture

Any even number greater than 2 can be expressed as the sum of two prime numbers.

Conjecture of twin primes

There are infinite prime number pairs (p, p+2).

2. Proof Process

Proof:

First step: Prove that any even number greater than or equal to 2 can be expressed as the difference of two prime numbers.

Construct a sequence R that only contains prime numbers and sets all other numbers to 0; R(0)=0, R(1)=0.

R=[0,0,0,3,0,5,0,7,0,0,0,11, ...]

To prove the above proposition, it can be formulated as follows: if the sequence R is shifted right by M positions and the resulting sequence has the same prime numbers at the same positions as the original sequence R

R(t) & R(t+M) = 1 (t∈（R(t) ≠0)).

At this point, the even number M is the difference between the two prime numbers at the same position in the shifted sequence and the original sequence R.

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Now using the method of proof by contradiction：

Let's assume that there is an even number greater than or equal to 2 that cannot be expressed as the difference of two prime numbers, let's call it N. In this case, all other even numbers can be expressed in this way.

R(t)&R(t+2)=1
R(t)&R(t+4)=1
R(t)&R(t+6)=1
...
R(t)&R(t+N+2)=1
R(t)&R(t+N+4)=1
R(t)&R(t+N+6)=1
...
R(t)&R(t+N+N)=1
……

Each of the equations can find a number to satisfy when t takes values of R(t) ≠ 0.

Below we prove that the above conditions R(t) & R(t+N) = 0 and R(t) & R(t+N+2) = 1, R(t) & R(t+N+4) = 1, R(t) & R(t+N+6) = 1, ... are impossible!

Because R(t+S+N) is the subsequence of R(t+N);

Then

R(t)&R(t+N)=0
...
R(t)&R(t+2+N)=0
R(t)&R(t+4+N)=0
R(t)&R(t+6+N)=0
...
R(t)&R(t+N+N)=0
……

As shown above, If an even number N cannot be represented as the difference of two prime numbers, then any even number greater than it cannot be represented as the difference of two prime numbers.

Second step: Prove that any even number greater than 2 can be expressed as the sum of two prime numbers.

Construct a sequence R that only contains prime numbers and sets all other numbers to 0; R(0)=0, R(1)=0.

R=[0,0,0,3,0,5,0,7,0,0,0,11, ...]

To prove the above proposition, it can be formulated as follows: if the sequence R is rotated and shifted right by M positions, the resulting sequence has the same prime numbers at the same positions as the original sequence R

R(t) & R(-t+M) = 1 (t∈（R(t) ≠0),R(-t)=R(t)).

In this case, the even number M is the sum of the two prime numbers that are both prime at the same position in the rotated and translated sequence and the original sequence R.

Now using the method of proof by contradiction：

Let's assume that there is an even number greater than 2 that cannot be expressed as the sum of two prime numbers, let's call it N. In this case, all other even numbers can be expressed in this way.

R(t)&R(-t+N-2)=1
R(t)&R(-t+N-4)=1
R(t)&R(-t+N-6)=1
...
R(t)&R(-t+N-N)=1
……

Each of the equations can find a number to satisfy when t takes values of R(t) ≠ 0.

Below we prove that the above conditions R(t) & R(-t+N) = 0 and R(t) & R(-t+N-2) = 1, R(t) & R(-t+N-4) = 1, R(t) & R(-t+N-6) = 1, ... are impossible!

Because R(-t-S+N) is the subsequence of R(-t+N);

Then

R(t)&R(-t+N)=0
...
R(t)&R(-t-2+N)=0
R(t)&R(-t-4+N)=0
R(t)&R(-t-6+N)=0
...
R(t)&R(-t-N+N)=0
……

As shown above, If an even number N greater than 2 cannot be represented as the sum of two prime numbers, then any even number smaller than it cannot be represented as the sum of two prime numbers.

Third step: Prove that there are infinite prime number pairs (p, p+2).

Here, we also using the method of proof by contradiction**.** Let's assume that there are only finite prime number pairs (p, p+2). Let U be a prime number greater than the largest prime pair p+2, and n∈（R(n) ≠0). At this point, we construct the sequence R=[U,0,…] as in the first step. Since there are no more prime number pairs (p, p+2), the sequence R shifted right by 2 will not have any prime numbers at the same positions as the original sequence R. According to the derivation in the **First step**：

R(n)&R(n+2)=0
R(n)&R(n+4)=0
R(n)&R(n+6)=0
…

It can be reformulated as:
shifting right by 2,
R(n)&R(n+2)=0
R(n)&R(n+2+2)=0
R(n)&R(n+4+2)=0
……
shifting right by 4,
R(n)&R(n+4)=0
R(n)&R(n+2+4)=0
R(n)&R(n+4+4)=0
……
shifting right by 6,
R(n)&R(n+6)=0
R(n)&R(n+2+6)=0
R(n)&R(n+4+6)=0
……

The prime numbers greater than U will necessarily appear at the even positions after U. However, as we have deduced above, no matter how much the sequence R is shifted right, there will never be prime numbers at the same positions as the original sequence R. This means that there are only finite prime numbers!

However, Euclid has already proved that there are infinite prime numbers, so the above assumption is incorrect. In other words, there still exist prime number pairs (p, p+2) after the prime number U, so there are infinite prime number pairs (p, p+2).

Fourth step: Generalize the above reasoning to (p, p+2k) based on the above derivation process.

Here, we also using the method of proof by contradiction**.** Let's assume that there are only finite prime number pairs (p, p+2k). Let U be a prime number greater than the largest prime pair p+2k, and v∈（R(n) ≠0). At this point, we construct the sequence R=[U,0,...] as in the first step. Since there are no more prime number pairs (p, p+2k), the sequence R shifted right by 2k will not have any prime numbers at the same positions as the original sequence R. According to the derivation in the **First step**：

$R(n)\&R(n+2k)=0$
$R(n)\&R(n+2k+2)=0$
$R(n)\&R(n+2k+4)=0$
…

It can be reformulated as:
shifting right by 2,
$R(n)\&R(n+2k+2)=0$
$R(n)\&R(n+2k+2+2)=0$
$R(n)\&R(n+2k+4+2)=0$
……
shifting right by 4,
$R(n)\&R(n+2k+4)=0$
$R(n)\&R(n+2k+2+4)=0$
$R(n)\&R(n+2k+4+4)=0$
……
shifting right by 6,
$R(n)\&R(n+2k+6)=0$
$R(n)\&R(n+2k+2+6)=0$
$R(n)\&R(n+2k+4+6)=0$
……

The prime numbers greater than U will necessarily appear at the even positions after U. However, as we have deduced above, no matter how much the sequence R is shifted right, there will never be prime numbers at the same positions as the original sequence R. This indicates that there are only finite prime numbers!

However, Euclid has already proved that there are infinite prime numbers, so the above assumption is incorrect. In other words, there still exist prime number pairs (p, p+2k) after the prime number U, so there are infinite prime number pairs (p, p+2k).

3. Conclusion

Based on the above proofs, we can draw the following conclusions:

1. Any even number greater than or equal to 2 can be expressed as the difference of two prime numbers; If an even number cannot be expressed as the difference of two prime numbers, then all even numbers greater than it cannot be expressed as such.

2. Any even number greater than 2 can be expressed as the sum of two prime numbers; If an even number greater than 2 cannot be expressed as the sum of two prime numbers, then all even numbers less than it cannot be expressed as such.

3. There are infinite prime number pairs (p, p+2); If there are a finite number of prime pairs (p, p+2), then the number of prime numbers is finite.

4. There are infinite prime number pairs (p, p+2k); If there are a finite number of prime pairs (p, p+2k), then the number of prime numbers is finite.