Four-Dimensional Newtonian Relativity

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Abstract

The constancy of the speed of light seems to imply that time and space are not absolute. However, we aim to demonstrate that this is not necessarily the case. In this article, we use Newton's concepts of absolute time and absolute space, along with the hypothesis that physical space is four-dimensional, to construct an alternative formulation to the theory of special relativity. We prove this formulation is mathematically equivalent to Einstein's theory by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional Euclidean space.

I. Introduction

According to Newton, time and space are absolute [1]. That means time and space exist independently of physical events and each other. Furthermore, Newton argued that an object is at absolute rest if it is stationary with respect to absolute space or in absolute motion if it is moving with respect to absolute space [2]. For this reason, he contended that absolute space is a privileged frame of reference [3]. If Newton is correct, then the Galilean transformation should be the set of equations that accurately relate the time and space coordinates of two systems moving at constant velocity relative to each other [4]. The constancy of the speed of light led Einstein to conclude that Newton's views are wrong. However, it can be shown that that conclusion is not necessarily true. In this article, we will construct an alternative formulation to the theory of special relativity based on the postulates that time and space are absolute and the hypothesis that physical space is four-dimensional. We will prove the mathematical validity of this formulation by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional Euclidean space.

II. Postulates

The alternative formulation to the theory of special relativity that we propose is based on the following postulates:

- Time and space are absolute.
- Space is four-dimensional.
- Every object always moves at the speed of light with respect to absolute space.

The first postulate refers to the same concepts defined by Newton in 1687. The second postulate states our fundamental hypothesis: physical space is a four-dimensional Euclidean space. The third postulate posits that objects are never at rest with respect to absolute space and move only at one speed with respect to it—the speed of light. This proposition is similar to the one obtained from the theory of relativity, which asserts that all objects move through spacetime at the speed of light. The difference between these three postulates and five-dimensional space-time theories, such as Nordström's electromagnetic-gravitational theory [5] and the Kaluza-Klein theory [6–8], is that those formulations do not consider time and space to be absolute, nor do they treat space as Euclidean.

The mathematical formalism of these postulates is the following:

- The coordinates of two systems that move at a constant velocity relative to each other are related by the transformations of the Galilean group (any composition of uniform motions, translations and rotations in four-dimensional Euclidean space [9]).
- Five equations are required to relate the coordinates between two inertial frames of reference (four equations for the spatial coordinates and one equation for the temporal coordinate).
- The speed between any inertial frame of reference that represents a physical object and absolute space must be equal to the speed of light.

In addition to suggesting postulates about the nature of time and space, we need to take into account that the theory of relativity is grounded on the presumption that space is three-dimensional. This remark can be stated as follows:

- Special relativity presupposes that space has three dimensions, but if space actually has four dimensions, then that erroneous assumption would have affected the mathematical formulation of the theory.

We shall refer to this statement as the presupposition principle. This principle is inherently different from the aforementioned postulates because it does not describe the physical world. Instead, it highlights the possibility that Einstein could have derived the Lorentz transformation under the wrong assumption about the dimensionality of space.

Let's take a closer look at Einstein's formulation of special relativity [10]. In the third section of his article "On the Electrodynamics of Moving Bodies," Einstein describes two orthogonal coordinate systems: K and k. Each system contains three axes perpendicular to one another. The axes along the X direction of the two systems coincide, while the axes along the Y and Z directions are parallel. The origin of k moves relative to K at a constant velocity v in the X direction. The coordinates x, y, z and t specify the place and time of an event according to K, whereas the coordinates ξ, η, ζ and τ specify the place and time of the same event according to k. Now, if space is indeed four-dimensional, then it is clear that Einstein did not represent the fourth rectangular component of the position vectors in his formulation (implicitly assigning them a value of zero). Furthermore, in that same section, he set τ as a function of x, y, z and t. However, if time is absolute, then τ cannot be a function of the position coordinates. Thus, we can conclude that Einstein's presumption about the dimensionality of space affected his formulation in two ways:

- He assumed that only four equations (instead of five) are required to relate the coordinates between two inertial frames of reference (three equations for the spatial coordinates and one equation for the temporal coordinate). The equation he discarded was the one that represents that time is absolute.
- He implicitly assigned values of zero to the fourth spatial coordinates of physical events.

In the next section, we will show how the Lorentz transformation emanates from these two statements.

The three postulates presented here, in conjunction with the presupposition principle, constitute the four-dimensional Newtonian formulation of special relativity. We want to emphasize that the postulates are proposed as a description of reality, whereas the presupposition principle is offered as an explanation of where the Lorentz transformation emerges from. Consequently, this formulation contains two sets of equations: the first one gives a mathematical description of the physical world as it actually is (the Galilean transformation for frames of reference in four-dimensional Euclidean space), while the second one arises from the presumption that space is three-dimensional (the Lorentz transformation). It is the latter set of equations that is mathematically equivalent to the theory of special relativity.

III. Derivation

In this section, we will prove that the four-dimensional Newtonian formulation of special relativity is mathematically equivalent to Einstein's theory. To do this, we will use four rectangular coordinate systems: S, A, A′ and S′. Each system contains four coordinates that specify the position of physical events in four-dimensional Euclidean space and a time coordinate that specifies the instant in which those events take place. Hence, the coordinates of an arbitrary event E are

- $(x_1,\, x_2,\, x_3,\, x_4,\, t)$ according to S
- $(X_1,\, X_2,\, X_3,\, X_4,\, T)$ according to A
- $(X_1',\, X_2',\, X_3',\, X_4',\, T')$ according to A′
- $(x_1',\, x_2',\, x_3',\, x_4',\, t')$ according to S′

The Lorentz transformation that we will derive is the Lorentz boost in the x_1 direction. We will derive it from a composition of three transformations of the Galilean group (two uniform motions and a rotation in four-dimensional Euclidean space). The configuration of that composition is the following: The coordinate systems A and A′ are fixed with respect to absolute space, and their axes are rotated according to

$$X_1 = X_1' \, cos\, \theta + X_4' \, sin\, \theta \tag{1}$$

$$X_2 = X_2' \tag{2}$$

$$X_3 = X_3' \tag{3}$$

$$X_4 = -X_1' \, sin\, \theta + X_4' \, cos\, \theta \tag{4}$$

$$T = T' \tag{5}$$

where θ is the angle of rotation (a constant value between -90° and 90°). The coordinate system S represents an inertial frame of reference. It moves along the common axis X_4–x_4. According to the postulates we propose, inertial frames of reference move at the speed of light with respect to absolute space. Therefore, the Galilean transformation equations are

$$X_1 = x_1 \tag{6}$$

$$X_2 = x_2 \tag{7}$$

$$X_3 = x_3 \tag{8}$$

$$X_4 = x_4 + ct \tag{9}$$

$$T = t \tag{10}$$

where c is the speed of light. Likewise, the coordinate system S′, which also represents an inertial frame of reference, moves at the speed of light along the common axis X_4'–x_4'. Thus, the Galilean transformation equations for this case are

$$X_1' = x_1' \tag{11}$$

$$X_2' = x_2' \tag{12}$$

$$X_3' = x_3' \tag{13}$$

$$X_4' = x_4' + ct' \tag{14}$$

$$T' = t' \tag{15}$$

These equations (1–15) completely specify the configuration of our system. Our objective now is to find a mathematical relation between S and S′ that contains the velocity component of S′ with respect to S along the x_1 axis. We start by substituting equations 6–15 into equations 1–5:

$$x_1 = x_1' \, cos\,\theta + \left(x_4' + ct'\right) \, sin\,\theta \tag{16}$$

$$x_2 = x_2' \tag{17}$$

$$x_3 = x_3' \tag{18}$$

$$(x_4 + ct) = -x_1' \, sin\,\theta + \left(x_4' + ct'\right) \, cos\,\theta \tag{19}$$

$$t = t' \tag{20}$$

Then we use equation 20 to interchange t and t' in equations 16 and 19:

$$x_1 = x_1' \, cos\,\theta + (x_4' + ct) \, sin\,\theta \tag{21}$$

$$x_2 = x_2' \tag{22}$$

$$x_3 = x_3' \tag{23}$$

$$\left(x_4 + ct'\right) = -x_1' \, sin\,\theta + \left(x_4' + ct\right) \, cos\,\theta \tag{24}$$

$$t = t' \tag{25}$$

The velocity $\vec{v}$ of S′ relative to S is

$$\vec{v} = v_1 \, \hat{x}_1 + v_4 \, \hat{x}_4 \tag{26}$$

$$v_1 = \frac{dx_1}{dt} = c \, sin\,\theta \tag{27}$$

$$v_4 = \frac{dx_4}{dt} = c \, \left(cos\,\theta - 1\right) \tag{28}$$

where v_1 is the velocity component of S′ with respect to S along the x_1 axis (derived from equation 21), v_4 is the velocity component of S′ with respect to S along the x_4 axis (derived from equations 24 and 25), $\hat{x}_1$ is the unit vector along the x_1 axis and $\hat{x}_4$ is the unit vector along the x_4 axis. The derivatives of x_1' and x_4' with respect to t are zero because the velocity of S′ with respect to itself is equal to zero. This result is useful because equation 27 allows us to write $sin\,\theta$ in terms of v_1:

$$sin\,\theta = \frac{v_1}{c} \tag{29}$$

The Pythagorean trigonometric identity tells us that

$$cos\,\theta = \sqrt{1 - sin^2\,\theta} \tag{30}$$

Hence, we can also write $cos\,\theta$ in terms of v_1 by substituting equation 29 into equation 30:

$$cos\,\theta = \sqrt{1 - v_1^2/c^2} \tag{31}$$

The gamma factor for the Lorentz boost in the x_1 direction is given by

$$\gamma \equiv \frac{1}{\sqrt{1 - v_1^2/c^2}} \tag{32}$$

Consequently, we have that

$$cos\,\theta = \frac{1}{\gamma} \tag{33}$$

Lastly, we substitute equations 29 and 33 into equations 21 and 24. Therefore, we can rewrite equations 21–25 the following way:

$$x_1 = \frac{x_1'}{\gamma} + \left(x_4' + ct\right)\ \frac{v_1}{c} \tag{34}$$

$$x_2 = x_2' \tag{35}$$

$$x_3 = x_3' \tag{36}$$

$$\left(x_4 + ct'\right) = -x_1'\,\frac{v_1}{c} + \frac{\left(x_4' + ct\right)}{\gamma} \tag{37}$$

$$t = t' \tag{38}$$

Equations 34–38 provide the mathematical relationship between S and S′ that we were looking for. According to the postulates we propose, they represent the physical reality of the system (describing events that occur in four-dimensional Euclidean space). Now we proceed to show how the Lorentz transformation emerges from applying the presupposition principle to this set of equations.

As stated in the previous section, Einstein's presumption about the dimensionality of space made him conclude that only four equations are required to relate the coordinates between two inertial frames of reference. The equation he discarded was the one that states that time is absolute. For this reason, we discard equation 38 from the set of equations 34–38. This gives

$$x_1 = \frac{x_1'}{\gamma} + \left(x_4' + ct\right)\ \frac{v_1}{c} \tag{39}$$

$$x_2 = x_2' \tag{40}$$

$$x_3 = x_3' \tag{41}$$

$$\left(x_4 + ct'\right) = -x_1'\,\frac{v_1}{c} + \frac{\left(x_4' + ct\right)}{\gamma} \tag{42}$$

Einstein's assumption also led him to implicitly assign values of zero to the fourth spatial coordinates of physical events. Mathematically this means that

$$x_4 = 0 \tag{43}$$

$$x_4' = 0 \tag{44}$$

Substituting equations 43 and 44 into equations 39–42 gives

$$x_1 = \frac{x_1'}{\gamma} + v_1 t \tag{45}$$

$$x_2 = x_2' \tag{46}$$

$$x_3 = x_3' \tag{47}$$

$$ct' = -\frac{v_1}{c}x_1' + \frac{ct}{\gamma} \tag{48}$$

The coordinate t' needs to be written in terms of the coordinates x_1 and t. Hence, we solve for x_1' in equation 45 and substitute it into equation 48 (Appendix). After doing this, we get

$$x_1 = \frac{x_1'}{\gamma} + v_1 t \tag{49}$$

$$x_2 = x_2' \tag{50}$$

$$x_3 = x_3' \tag{51}$$

$$ct = \frac{v_1}{c}x_1 + \frac{ct'}{\gamma} \tag{52}$$

The last step of the derivation is to solve for the coordinates x_1', x_2', x_3' and t' in equations 49–52, respectively:

$$x_1' = \gamma(x_1 - v_1 t) \tag{53}$$

$$x_2' = x_2 \tag{54}$$

$$x_3' = x_3 \tag{55}$$

$$t' = \gamma\left(t - \frac{v_1 x_1}{c^2}\right) \tag{56}$$

Equations 53–56 correspond to the Lorentz transformation of inertial frames of reference that move relative to each other at a constant velocity v_1 along their common x_1–x_1' axis (also known as the Lorentz boost in the x_1 direction). Thus, we have proven that the Lorentz transformation can be derived from the Galilean transformation for frames of reference in four-dimensional Euclidean space. The more general form of the Lorentz transformation can be obtained by extending the procedure presented in this article.

As a final note, we want to emphasize that the Galilean transformation derived in this section (given by equations 34–38) describes a single event. However, when equation 38 is discarded and the mathematical conditions of the presupposition principle are imposed, then the resulting equations describe two events that occur at the same place in absolute space but at different times (one event occurs at the instant when $x_4 = 0$, while the other event occurs at the instant when $x_4' = 0$). That would be the interpretation of this result from a mathematical perspective. From a physical perspective, this result tells us that the effects from the Lorentz transformation (such as time dilation, length contraction and the constancy of the speed of light) are actually depth perception effects that are being interpreted as real effects because the fourth spatial dimension is not being taken into account. We will address these remarks more profoundly in a future paper.

IV. Conclusion

An alternative formulation to the theory of special relativity was constructed from the concepts of absolute time and absolute space, as defined by Newton, and from the hypothesis that physical space is four-dimensional. The formulation contains two sets of equations: the first one describes the physical world as it is, while the second one emerges from the wrong assumption about the dimensionality of space. The second set of equations (the Lorentz transformation) is mathematically equivalent to Einstein's theory. However, the interpretation of those equations is significantly different. The four-dimensional Newtonian formulation of special relativity interprets the effects predicted by the Lorentz transformation as depth perception effects, whereas special relativity interprets those effects as being real. Therefore, four-dimensional Newtonian relativity proves that the constancy of the speed of light is not necessarily incompatible with the concepts of absolute time and absolute space. Furthermore, and perhaps more significantly, the result presented here could be considered mathematical evidence that physical space is four-dimensional.

Dedication

This article is dedicated to the memory of my father, Dr. Lorenzo León Callender López, who always supported me and was there for me. Without him, this work would not have been possible.

Appendix

In this appendix, we provide the steps missing from our derivation of the Lorentz transformation. First, we solve for x'_1 in equation 45

$$x'_1 = \gamma\left(x_1 - v_1 t\right)$$

and then we substitute it into equation 48

$$ct' = -\frac{v_1}{c}x'_1 + \frac{ct}{\gamma}$$

$$ct' = -\frac{v_1}{c}\gamma\left(x_1 - v_1 t\right) + \frac{ct}{\gamma}$$

$$\frac{ct'}{\gamma} = -\frac{v_1}{c}\left(x_1 - v_1 t\right) + \frac{ct}{\gamma^2}$$

The square of the gamma factor (equation 32) is given by

$$\gamma^2 = \frac{1}{1 - v_1^2/c^2}$$

Thus we have that

$$\frac{ct'}{\gamma} = -\frac{v_1}{c}\left(x_1 - v_1 t\right) + ct\left(1 - \frac{v_1^2}{c^2}\right)$$

$$\frac{ct'}{\gamma} = -\frac{v_1}{c}x_1 + \frac{v_1^2}{c}t + ct - \frac{v_1^2}{c}t$$

$$\frac{ct'}{\gamma} = -\frac{v_1}{c}x_1 + ct$$

$$ct = \frac{v_1}{c}x_1 + \frac{ct'}{\gamma}$$

which is the result we were looking for (equation 52).

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