QCD Self-Consistent Only With a Self-Consistenct QED

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Abstract

The Standard Model of particle physics, based on the group structure $SU(N)_c \otimes SU(2)_L \otimes U(1)_Y$ $(for N_c = 3)$, has been very successful. However in it, the electric charge is not quantized and is fixed by hand to be 2/3 and -1/3. This is its major shortcoming. This model runs into conflict with another similarly structured, but actually quite different model, wherein the electric charge is fully quantized and depends upon colour degree of freedom as well. We study this basic conflict between these models and how they connect to a consistent study of Quantum Chromodynamics (QCD) for arbitrary number of colours. We run into a basic issue of consistency of Quantum Electrodynamics (QED) with these fundamentally different charges. Study of consistency of (QCD + QED) together, makes discriminating and conclusive statements about the relevance of these two model structures.

Keywords: QCD, QED, electric charge, Standard Model, spontaneous symmetry breaking

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The Standard Model (SM) of particle physics, based on the group structure $SU(3)_c \otimes SU(2)_W \otimes U(1)_W$, has been a very successful model. However it has a well known shortcoming. It is that the definition of, as fundamental a quantity as the electric charge, is merely conventional and arbitrary in the SM. This reality is reflected by the fact that popularly there exist two independent and arbitrary definitions of the electric charge in the SM.

One commonly used definition of the electric charge in the SM is [1]:

$$Q = T_3{}^W + \frac{Y_W}{2} \tag{1}$$

The second equally popular definition is [2]:

$$Q = T_3{}^W + Y_W' \tag{2}$$

The value of the weak-hypercharges is put in by hand to ensure that for example the electric charges of the u- and the d- quarks are 2/3 and -1/3 respectively. As quarks carry colour and electric charge, both QCD and QED are simultaneously relevant for them. These fixed charges 2/3 and -1/3 in SM, manisfest themselves in the QED description of these quarks.

However, it has been shown that, in line with the full group stucture for arbitary number of colours $SU(N)_c \otimes\ SU(2)_L \otimes U(1)_Y$ (also for $N_c = 3$ as well), and having the same theoretical structure as the above electro-weak model, and with the same Englert-Brout-Higgs mechanism of spontaneous suymmetry breaking etc., one obtains a more general definition of the electric charges [3,4] as,

$$Q_u = Q_c = Q_t = \frac{1}{2}(1 + \frac{1}{N_c}) \tag{3}$$

$$Q_d = Q_s = Q_b = \frac{1}{2}(-1 + \frac{1}{N_c}) \tag{4}$$

Thus this model the electric charge of quarks intrinsically know of the colour degree of freedom. It was also shown that the correct charge does not merely take the static values of 2/3 and -1/3 (i.e., independent of any colour), but can differ from these values for arbitrary number of colours [3].

Thus we have two different electric charge definitions - one rigid and non-colour dependent 2/3 and -1/3 charges in the SM, and another one with intrinsic colour dependence as given above in eqns. (3) and (4). Does the fact, that both of these have the same value of charge for three colours, make them physically indistinguishable from each other? Actually it is not so.

The $\pi^0 \to \gamma\gamma$ decay rate is [1,2]

$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = N_c^2 (Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_{\pi 0}^2}{64\ \pi^3 F_\pi^2} \quad (5)$$

where Q_u and Q_d are the $u-$ and $d-$ quark charges, m_{π^0} is the neutral pion mass, $\alpha = \frac{e^2}{4\pi}$ and F_π, the pion decay constant $\sim$ 91MeV.

Experimentally the decay rate is $\sim$ 7.8 eV [1,2]. With SM static charges of $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$, and with no colours ($N_c = 1$), one obtains the decay rate of 0.84 eV. This is much too low a value. One is therefor forced to include $N_c = 3$ to fit the experimental value. This is taken as a standard proof of the evidence of 3-colours in particle physics [1, p. 182 ; 2. p. 368].

Next what has our colour dependent charges (eqns. (3) and (4)) have to say about it. The colour factors in eqn. (5) go as:

$$N_c{}^2 (Q_u{}^2 - Q_d{}^2)^2 = N_c{}^2 \left[\left\{ \frac{1}{2}\ \left(1 + \frac{1}{N_c}\right) \right\}^2 - \left\{ \frac{1}{2}\ \left(-1 + \frac{1}{N_c}\right) \right\}^2 \right]^2 = 1 \quad (6)$$

And hence overall there is no N_c- dependence left in the decay rate of $\pi^0 \to \gamma\gamma$ and the subsequent result matches the experiment well. So when correct colour dependent electric charges for quarks are taken, the decay rate is actually independent of colour degrees of freedom. Hence the above pion decay rate is not a proof of three colours [5,6].

Hence physics does discriminate between the two kind of charges in SM structure - colour-independent and colour-dependent definitions. The latter being a more general one, and is thus definitely a better result [5,6]. However these are clearly defining different kind of Quantum Electrodynamic stuctures. For both of these the QCD structure is obviously the same but intrinsic QED structure is definitely different.

Let us explore this in some detail. Study of QCD for arbitrary number of colours has been found to be a very revealing exercise about the basic nature of it as shown by 't Hooft [7] and by the following work done by Witten [8]. All this is telling us what the intrinsic structure of QCD is in reality.

These studies of QCD for arbitrary number of colours [8] shows, besides other interesting features, that for an arbitrary number of colours, baryon has a finite size and has a mass going as:

$$M(baryon) \sim N_c \tag{7}$$

These are amongst the most significant results of this analysis within QCD. This is part of what the intrinsic structure of QCD predicts. Put another way, these results arise from what self-consistency of QCD demands.

However Witten made a major assumption. He asssummed that he could ignore QED in his analysis as the electromagnetic force was so much weaker than the strong QCD force [8,9]. Thereafter he and others did extensive calculations within quark model for arbitrary number of colours [10].

In their model the baryons are made up of N_c number of quarks [10]. There proton has $\frac{N_c+1}{2}$ u-quarks and $\frac{N_c-1}{2}$ d-quarks; and vice versa for neutron. The proton and neutron charges were:

$$Q_p = \left(\frac{N_c+1}{2}\right) Q_u + \left(\frac{N_c-1}{2}\right) Q_d \tag{8}$$

$$Q_n = \left(\frac{N_c-1}{2}\right) Q_u + \left(\frac{N_c+1}{2}\right) Q_d \tag{9}$$

Now with their static charge values of 2/3 and -1/3 one gets:

$$Q_p = \frac{N_c+3}{6} \;\; ; \;\; Q_n = \frac{N_c-3}{6} \tag{10}$$

Hence proton and neutron charges are one and zero respectively only for $N_c = 3$. Both are non-integral or non-zero for arbirary number of colours. Now Witten et. al [10] had unfortunately neglected the fundamental Coulomb self-energy term contribution to the baryon masses in eqn. (7). But this should add as a QED contribution. And thus the QCD plus QED contributions to baryon mass are,

$$M(proton) \sim N_c + C\frac{\left(\frac{N_c+3}{6}\right)^2}{R} \tag{11}$$

wher C is a constant and R is the finite size of proton. Now the baryon mass is blowing as ${N_c}^2$ due to QED part. This is messing up the whole analsis based on self-consistent QCD only. Thus here QCD plus QED tells us that there are no stable large number of colour baryons. This is thus what the conventional SM is telling us.

The above crisis within QED is perhaps due to the use of the static charges 2/3 and -1/3 of the SM. Indeed this turns out to be true. Take our correct colour dependent charges in eqns. (3) and (4). Put these charges in eqns. (8) and (9) and lo and behold, the proton charge is always one and neutron is always neutral for any arbitrary colour. Thus the Coulomb self-energy term is always finite and indeed the baryon mass only goes as eqn.(7). And thus the prediction of the self-consistent analysis in QCD is honoured and retained within the new self-consistent structures arising in QED with the proper colour dependent charges.

Hence the colour-dependent charges give a good and correct descrption of QED which goes consistenly with the QCD. Thus it is QCD+QED (but with proper colour dependent charges) which provides a complete and consistent descrption of hadrons.

So we arrive at the important conclusion that the static charges of the SM lead to inconsistent QED structures. The new colour dependent charges give consistent QED description and which conjoins well with the proper and correct structure of QCD for arbitrary number of colours.

Now a few remarks about what was actually done when fully quantized and colour dependent charges were obtained [3,4]. Now the structure of the three generations was exactly the same as that in the SM. An Englert-Brout-Higgs field, exactly as in the SM, was also brought in. The symmetry was also broken spontaneously. The massses were also generated in the same manner as in the SM. But the major difference was, that in contrast to the SM (where the electric charges were arbitrarily pre-defined), the electric charge was fixed by the process of SSB itself. All the unknown hypercharges were relatively fixed due to the richness of structure provided by the cancellation of all the anomalies to ensure renormalizability of the theory. So in that picture not only did the masses arise from SSB but also the electric charges. Hence it is a much deeper physical picture which is emerging therein [3,4]. Thus this model goes beyond the confines of the conventional SM and thus may be called Quantized Charge Standard Model (QCSM). Note that as the electric charge knows of the colour degree of freedom, so there is already an unification of QCD with the electro-weak sector in the QCSM.

Here we emphasize that it is the QCSM which is providing an internally consistent QED picture to go along with a consistent picture of QCD to describe a consistent theory of hadrons for arbitrary number of colours.

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REFERENCES

1). T-P. Cheng and L-F. Li, "Gauge Theory of Elementary Particle Physics", Clarendon Press, Oxford, 1984

2). S. Weinberg, "The Quantum Theory of Fields", Vol. II, Cambridge University Press, Cambridge, 1996

3). A. Abbas, "Anomalies and charge quantization in the Standard Model with arbitrary number of colours", Phys. Lett. B 238, 344 (1990)

4). A. Abbas, "Spontaneous symmetry breaking, quantization of the electric charge and the anomalies", J. Phys. G 16, L163 (1990)

5). S. A. Abbas, S. A Sheikh, S Salahudin, M. Ilahi: researchgate.com: Abbas et al, On the Evidence of the Number of Colours in Particle Physics

6). S. A. Abbas, "Group Theory in Particle, Nuclear, and Hadron Physics", CRC Press, London, 2016

7). G. 't Hooft, Nucl. Phys. B 72 (1975) 461

8). E. Witten, Nucl. Phys. B 160 (1979) 59

9). E. Witten, Physics Today, 7(33) (1980) 38

10). G. S. Adkins, C. R. Nappi, E. Witten, Nucl. Phys. B 228 (1983) 552