STRESS-TESTING MODEL FOR CORPORATE BORROWER PORTFOLIOS.

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Abstract Despite the significant attention to the stress-testing issues in finances world-wide, the ways of quantitative assessment of the stress impact on the portfolios of non-public (in the absence of equity or debt market quotes) corporate borrowers are currently not sufficiently developed or standardized. The aim of this article is to propose high-level universal requirements to the quantitative models of stress-testing of non-public corporate borrower portfolios, and to describe the model, developed by the authors, which meets such requirements. Details of the model's calibration, implementation (using Monte-Carlo simulations) and some practical issues are covered in the article.

Keywords: stress-testing, credit risk, quantitative risk assessment

Introduction

Stress-testing has become one of the most important risk-management instruments worldwide. Despite the increasing interest in this subject, currently the problem of constructing stress-testing models for credit portfolios of non-public companies (further – stress-testing models) is covered by research and regulatory papers only fragmentarily and usually at a very high-level. Therefore, the main goals of this article are to formulate clear overall requirements for quantitative stress-testing models, and to propose one of the possible practical implementations of those requirements based on the modification of the Vasicek model – the model that underpins current international capital requirements (IRB approaches of Basel II-III).

According to our view, a quantitative stress-testing model for a portfolio of non-public corporate borrowers should fulfill the following requirements:

1. The approach should not be based only on default event modeling, but the model should also produce estimates of the changes in the portfolio rating structure. This will allow us to estimate potential losses (due to defaults) and RWA-changes (rating migrations) simultaneously and consistently.
2. Historical experience shows that concentration of credit risk in asset portfolios has been one of the major causes of bank distress; therefore the model should take into account concentration risks and correlation between default events.
3. The model should be based on the functional dependence between the defaults and dynamics of macro-variables. This property will allow us to model both potential losses based on real historical experience and losses based on hypothetical but plausible scenarios (produced by macro forecasters). Moreover, this property extends the scope of possible validation procedures, because the model could estimate losses during stress as well as expansion scenarios of economic development.
4. The model should allow us to estimate the marginal contribution of a single borrower to the stress-test results. Therefore, we could determine particular borrowers that are the main source of losses in a stress environment (potentially, it could be taken into account during risk-based pricing).
5. The approach should be universal; for example, it should allow us to make a consistent and transparent transformation of the stress-testing model into a portfolio model. This property will allow us to make a consistent comparison between stress-testing results and economic capital estimates. Moreover, it significantly reduces model development team efforts and increases the scope of possible validation procedures.

Methodology

Modification of the Vasicek model

We propose one of possible implementations of stress-testing for the credit portfolios of corporate borrowers (further - the Model), which is based on Monte-Carlo simulations and the modified Vasicek model (Vasicek, 1987). As will be shown, the Model meets all of the criteria described above.

The single systemic factor Vasicek model is based on the assumption that assets of the companies have two drivers – idiosyncratic (determining the individual properties of each company) and systemic (the overall macroeconomic environment). The change in assets of company A_i, according to the Vasicek model, is equal to the sum of two normally distributed random variables: idiosyncratic – ε_i and systematic Z; the level of the dependence of the borrower from the systematic factor is captured by the correlation coefficient ρ_i:

$$A_i = \varepsilon_i \cdot \sqrt{1-\rho} + Z \cdot \sqrt{\rho} \quad (1)$$

If the value of company assets becomes less than some threshold level default occurs. Usually, the default threshold is defined as a company's debt burden. The default threshold could be calibrated based on the assumption of the normal distribution of asset return values A_i and a given borrower's default probability:

$$P\left(A_i < Ths_i\right) = PD_i \Rightarrow N\left(Ths_i\right) = PD_i \Rightarrow Ths_i = N^{-1}(PD_i)$$

We propose the following modification of the Vasicek model for stress-testing purposes: The default threshold should be decomposed on the sum of the components, each component consisting of the macro-variable M_j multiplied by coefficient β_{ij}, which defines the degree of dependence between default frequency and macro-variable j.

$$Ths_i = \alpha_i + \sum_{j=1}^{m} \beta_{ij} \cdot M_j \quad (2)$$

The proposed threshold decomposition will allow us to capture historical dependence between defaults and macro-variables, which also serves as a default correlation transmitter due to asset value dependence on the same factors. At the same time, the model contains explicit default correlation parameter ρ_i, by which we take into account the default correlation, which is not detectable through dependence on macro factors.

It is impossible to statistically identify dependence between macro-variables and individual borrowers; therefore companies should be grouped into subsets with similar risk characteristics - rating classes.

It is very important to choose macro-factors for model calibration correctly. As general recommendations, we propose the following selection criteria:

1. Each macro-variable should have significant individual predictive power regarding historical default frequencies (R^2).
2. The correlation between selected macro-variables should be relatively low (for purposes of model stability).

A high correlation between all predictive macro-variables is common for emerging economies (for example, the price of oil in OPEC countries or Russia is the main economic driving force); therefore in order to fulfill the second requirement it is recommended to replace the original dynamics of the macro-variables M_i by the principle components of macro-variables M_i with zero correlation between them.

According to the proposed modifications, the density function for default frequency could be written as:

$$f(x) = \prod_{q=1}^{Q} \binom{n_q}{x_q} \int_0^1 \prod_{i=1}^{R} (N(\frac{\alpha_i+\sum_{j=1}^{m} \beta_{ij} \cdot M_{qj} + \sqrt{\rho_i} \cdot Z}{\sqrt{1-\rho_i}}))^{x_q} (1 - N(\frac{\alpha_i+\sum_{j=1}^{m} \beta_{ij} \cdot M_{qj} + \sqrt{\rho_i} \cdot Z}{\sqrt{1-\rho_i}})^{n_q - x_q} \, dN(Z) \quad (3),$$

where

Q – index of the time period (quarter or year).
n_q – number of borrowers in the portfolio during the period q.
x_q – number of defaults during the period q.
N – normal distribution function.
M_{qj} – historical value of j macro-variable during the period q.
R – number of rating classes.

Given the default density function, information of the historical default frequencies by rating classes and historical values of macro-variables, parameters α_i, β_{ij}, ρ_i could be found using the maximum likelihood approach. As a result, we could produce conditional PDs for rating classes given the macro-forecast.

Monte-Carlo simulation schema

One of the key requirements for the stress-testing model is the ability to estimate changes in the rating structure of the portfolio over time. The most obvious approach for this task is to incorporate migration matrixes into the model. Due to the dependence of the rating migration dynamics on the economic cycle, it is recommended to use different migration matrixes for stress and expansion scenarios.

We propose the following Monte-Carlo simulation schema, which takes into account the proposed density function (3) and migration matrixes:

1. For the given macro-variable dynamics (from the macro-forecast) for the stress-testing period, conditional PDs are calculated (using (2)) for each rating class - Ths_i.
2. The normal random variable Z is generated (systemic factor).
3. The normal random variable ε_i is generated for each borrower in the portfolio (idiosyncratic factor).

4. If $\varepsilon_i \cdot \sqrt{1-\rho} + Z \cdot \sqrt{\rho} \leq Ths_i$, a default event is fixed for a borrower during a current period. A defaulted borrower is excluded from the portfolio, and its exposure multiplied by LGD is added to the total portfolio losses within the scenario.
5. If the borrower does not default, its ratings for the next period are changed in accordance with the migration matrix – a uniformly distributed random number is generated $r \in [0; 1]$, and a new rating for the next period is assigned to the borrower according to the probabilistic interval of the migration matrix in which random number r falls.
6. Items 1-5 are repeated until the required forecast horizon is achieved.

The result of MC simulations is an array of losses. This array is a numerical representation of the density function of losses due to borrowers defaulting. On the basis of this distribution, the mean and quantiles of portfolio losses can be estimated.

The marginal contribution of individual borrowers to the stress-test results can be estimated using an approach similar to the Monte-Carlo model, which is described, for example, in (Tasche, 2000).

Transformation to a portfolio model

The proposed stress-testing model could be easily transformed into a portfolio model (the model dedicated to the estimation of unexpected losses). In the case of a portfolio model, a macro-forecast should be excluded from the model by replacing the forecasted M_{qi} values by the random values M_{qi}. The distribution function of M_{qi} could be calibrated using the historical values of macro-variables.

One of the most flexible approaches that could capture the time evolution of macro-variables is the ARIMA model. The ARIMA model would capture the following aspects of time evolution of macro-variables:

1) Stationary part:
    a. Long-term trends.
    b. Auto regression dependence (previous values of macro-variable dynamic influence values for the current period).
    c. Deviations from trends (prior to the period, error affects the current period's errors).
2) The random component - normally distributed random variables with a zero mean and covariance matrix (estimated on the basis of historical deviations of the real values of the macro factors from the ARIMA model).

In the case of the portfolio model, the Monte Carlo simulation schema should be modified in the following way:

1) Using the ARIMA model, M_{qi} values are generated for the estimation period.

2) The MC algorithm for the stress-testing model is started, in which, instead of forecasted macro-variables M_{qi}, random variables M_{qi} are used.

Conclusion

The proposed model meets all of the requirements mentioned in this article's introduction. The model could produce estimates both of losses due to borrowers' defaults and changes in the rating structure. The model is based on the functional dependence between dynamics of macro-variables and defaults; therefore it could be calculated for baseline and stress-scenarios. Comparison between the results in different scenarios will give us estimates of the changes of direct losses (defaults) and RWA changes (rating structure) due to stress events. The model could also be easily extended to the credit VAR model; therefore a bank could make consistent comparisons between stress-testing results and unexpected losses.

References

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